PROBABILISTIC DOMINANCE AND STATUS QUO BIAS

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Abstract. Decision makers have a strong tendency to retain the status quo. This well-documented phenomenon is termed status-quo bias. In many complicated choice problems governed by a status quo, decision makers consider alternatives that yield inferior outcomes in some criteria if they simultaneously yield improvements in a set of other criteria that is of relative importance. In an uncertain environment, we present the *probabilistic dominance* approach to status-quo bias – an alternative is considered acceptable to replace the status quo only if it yields a better outcome than the status quo with sufficiently high probability. Probabilistic dominance is applied and behaviorally characterized in a choice model that allows for a scope of biases towards the status quo, general enough to accommodate unanimity but also standard expected utility maximization. In addition, we show that the probabilistic dominance choice model predicts the well-known endowment effect and provide the means of calculating the buying and selling prices.

Keywords: Unanimity, probabilistic dominance, status quo bias, comparative status quo bias, endowment effect.

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1. Introduction

Standard choice models describe a decision maker who ought to choose one or a few elements from a collection of alternatives. These models typically describe methods that decision makers might use based on their utility, beliefs, ambiguity, information and alike. Payoff-irrelevant factors such as an initial endowment or a default reference point are considered irrelevant to the rational assessment of the alternatives and are frequently ignored. However, a growing amount of empirical data suggests that such factors affect behavior and that standard models are insufficient to describe real-life decision-making processes. It is well-established by now that individuals usually have a strong tendency to retain the current state of affairs. This phenomenon is traditionally referred to as *status quo bias* (Samuelson and Zeckhauser (1988)).

A recurrent explanation to status quo bias and related phenomena is anticipated regret. In particular, accumulated experimental findings point to a distinction between the status quo and new alternatives—sensation of regret is stronger when bad outcomes are a result of choosing a new alternative than when they result from retaining the status quo (e.g., Kahneman and Tversky (1982), Inman and Zeelenberg (2002) and Zeelenberg, van den Bos, van Dijk, and Pieters (2002)).

In the decision theoretic literature, a prominent approach involving status quo considerations is that of *unanimity*. According to this approach, an option is considered a candidate to replace the status quo if and only if it is considered better than the status quo with respect to every possible criterion. Examples include Bossert and Sprumont (2003), Masatlioglu and Ok (2005) and Ortoleva (2010). Criteria are determined subjectively by the decision maker according to the nature of the decision problem in study. For example, Bossert and Sprumont as well as Masatlioglu and Ok consider attributes of outcomes in a general framework, and Ortoleva (similar to Bewley (2002)) considers probability distributions over the state space.

The unanimity approach describes a decision maker who is boundedly rational in the sense that she is unable or unwilling (depending on the interpretation) to perform any sort of compromise relative to the status quo. In particular, she is willing to endure no regret upon replacement of the status quo with another alternative. However, behavior in complicated problems, such as accepting a new job offer or relocating to a different country, often does not conform with this approach. Decision makers may often allow for compromises in some criteria while simultaneous improvements in other criteria, that are subjectively important enough, are guaranteed. For instance, an assistant professor might take an associate professor position in a different state despite the uncertainty whether such a move will be successful or not, as long as she evaluates the chances of success to be sufficiently high.

Taking a choice-theoretic approach, the purpose of this paper is to present what we refer to as the *Probabilistic Dominance Model* of decision making under uncertainty—the decision maker considers an alternative choosable only if it yields a better outcome than the status quo with sufficiently high probability.

1.1. **Main results.** We adopt a setup similar to that in Masatlioglu and Ok (2005) but focus on inherent uncertainty as in Anscombe and Aumann (1963). In this set up, a choice problem is either a collection of acts, or a collection of acts and a status quo (which is an act) in this collection. In particular, it is assumed that the status quo is always a feasible alternative.[1] The decision maker is characterized by a choice correspondence that assigns a non-empty sub-collection of alternatives for each choice problem, with or without a status quo.

We present a set of axioms implying that the decision maker is associated with a subjective prior over the state space and two binary relations. The first admits an expected utility representation with respect to the subjective prior. The second admits a *probabilistic dominance* representation: one act is preferred to another if the probability (with respect to the same subjective prior) that the first act will return a better outcome than the second exceeds a certain threshold. The decision maker uses the following procedure to make her choices: whenever there is no status quo, the decision maker simply maximizes the first relation among all feasible alternatives. That is, she acts as a standard subjective expected utility maximizer. However, if the choice problem is governed by a status quo, she first applies probabilistic dominance considerations to eliminate all alternatives that do not return an outcome at least as good as the one returned by the status quo with high enough probability. She then maximizes the first relation over the collection of feasible acts that survived the elimination stage described

[1]See the discussion section for a simple extension of the model presented here to a more general one in which the status quo may not be feasible or exogenous.

above (note that this collection is never empty, as the status quo always survives the elimination stage).

To illustrate the proposed model, consider for instance the assistant professor example mentioned above. When contemplating the different offers, the assistant professor compares each offer to the one from the university she is currently employed at, trying to assess the quality of life she and her family would experience both from professional and personal aspects. Out of all offers, she is willing to consider only those that guarantee, with (subjectively) sufficiently high probability, a quality of life at least as good as the one she would experience in case she accepted the offer from her current working place. Then, out off all the offers she considers, she chooses the one that maximizes their expected well-being.

In the suggested decision model, the first step describes in what sense the decision maker distinguishes between the status quo and new alternatives (or, how she resolves her bias). She groups states to construct *decisive events*—events which are sufficiently significant for her to consider the alternatives that perform at least as well as the status quo within such events. This procedure conforms with the findings relating status quo bias to anticipated regret. Probabilistic dominance considerations guarantee that the probability the decision maker will regret having moved away from the status quo, once uncertainty is resolved, is not too high.[2,3] Alternatively, probabilistic dominance relations could be considered as ensuring some level of confidence when replacing the status quo. For example, the assistant professor would like to convince her spouse (or alternatively, herself[4]) that a particular offer is 'good enough' in the sense that the chance that it will improve their quality of life is significant.

[2]This procedure has a similar flavor as value at risk (VaR) from the finance literature, where investments yielding relatively high losses with sufficient probability are disregarded.

[3]Some experimental evidence for such comparisons can be found in Ritov (1996). In that study, when choosing between two alternatives, a major factor subjects exhibited was a concern to minimize the probability of regret. This interpretation of the first step of the suggested choice procedure raises a notion of regret different than those that had previously appeared in the literature, such as in Savage (1954), Bell (1982), Loomes and Sugden (1982) and Sarver (2008). All that matters for the decision maker is the probability of sensing regret.

[4]See the discussion in Shafir et al., (1993) page 33.

The scope of the probabilistic threshold in the representation allows for a range of biases towards the status quo. The higher the threshold, the more difficult it is for an act to survive the first stage of the decision making process. So, the higher the threshold parameter, the more biased towards the status quo the decision maker is. Note that the model is general enough to allow for no bias at all, so that choices follow a 'standard' maximization of expected utility (characterized by a threshold of 0), but also for a strong bias towards the status quo as expressed in the unanimity approach (which is characterized by a threshold of 1).

The chosen framework allows us, when comparing between the choices of two individuals, to present a definition of *revealing more bias towards the status quo*. The intuition behind the definition is the following. Assume that two decision makers, Ilse and Rick, both choose the same alternative given a status quo free choice problem. Assume in addition that at the same choice problem Rick no longer chooses this alternative once a status quo is present. If Ilse reveals more bias towards the status quo than Rick, then, in the presence of the same status quo, one would expect her to alter her choice as well. We study the implications of this definition to the suggested model. As a simple illustration, assume that both Ilse and Rick follow the probabilistic dominance choice model and that they share beliefs and tastes. Intuitively, Ilse reveals more bias towards the status quo than Rick if and only if Ilse's probabilistic threshold parameter is at least as high as that of Rick.

As an application of our choice model, we show that it predicts that the decision maker will demand more in order to give up a commodity than she would be willing to pay to acquire it. Thaler (1980) termed this well-known phenomenon the *endowment effect*.[5] We provide a simple way of calculating the buying and selling prices in terms of utility, and characterize by means of the representation those acts for which the decision maker exhibits the endowment effect.

1.2. **Related literature.** Status quo bias was first captured and termed by Samuelson and Zeckhauser (1988). Through laboratory and field experiments, they indicated the strong affinity of individuals to retain the alternative which is the status quo. This observation has lead to a significant number of empirical studies on the presence of status quo

[5]Additional examples are Knetsch and Sinden (1984) and Kahneman, Knetsch, and Thaler (1990).

bias (and related effects such as the endowment effect and reference-dependence) in important choices. Examples are 401(k) pension plans (Madrian and Shea (2001), Agnew, Balduzzi, and Sundén (2003) and Choi, Laibson, Madrian, and Metrick (2004)), electrical services (Hartman, Doane, and Woo (1991)) and car insurance (Johnson, Hershey, Meszaros, and Kunreuther (1993)).

These findings promoted the development of decision making models attempting to capture status quo effects. Tversky and Kahneman (1991) presented a reference-dependent choice model based on loss aversion. The intuition behind this model is that the status quo affects the utility of the decision maker in such a manner that relative losses loom larger than corresponding gains. Sugden (2003) and Munro and Sugden (2003) axiomatize reference-dependent preferences in the spirit of Tversky and Kahneman (1991)'s 'loss-aversion'. Köszegi and Rabin (2006) develop a reference-dependent preferences model, which adhere to loss-aversion as well, where the reference point is endogenously obtained out of, what they refer to as, personal equilibrium.

Masatlioglu and Ok (2005) and (2010) present a new approach to modeling status quo bias. Instead of affecting the underlying utility as in loss aversion, the presence of the status quo imposes mental constraints on what is choosable and what is not.[6] These constraints make some of the alternatives appear as inferior to the status quo and therefore unchoosable.[7] In Masatlioglu and Ok (2005), the status-quo bias axiom implies that the mental constraints admit a unanimity representation.[8,9]

Masatlioglu and Ok (2010) weaken the status-quo bias axiom and consider a much broader model. In particular, it is broader than the model presented here. For example, it is possible in the second stage of the process for the decision maker to consider some, *but not all*, alternatives that with high probability yield a good outcome relative to the

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[6]Rubinstein and Zhou (1999) consider a similar approach where the choosable alternatives are those closest to the status quo.

[7]See Masatlioglu and Ok (2010) for elaborated discussions on the differences between the two approaches.

[8]Formally, the mental constraints are associated with a transitive preference relation, where every such preference relation admits a unanimity representation.

[9]Sagi (2006) studies the implications of an axiom bearing the essence of the Masatlioglu and Ok (2005) status-quo bias axioms. The example given by Sagi, when considered in the present framework, describes an attitude towards the status-quo which is identical to unanimity.

status quo, as well as alternatives that yield a good outcome relative to the status quo with much lower probabilities.

Lastly, similarly to the current paper, Ortoleva (2010) studies the implications of status-quo bias in the presence of uncertainty. He adopts the Masatlioglu and Ok (2005) status-quo bias axiom and obtains a unanimity representation in the sense of Bewley (2002), where the criteria consist of different prior beliefs over the state space.

1.3. Organization.

In the following section we discuss the main model in detail, describing the framework in Section 2.1, introducing the representation in Section 2.2, the axioms in Section 2.3 and the representation theorem in Section 2.4. A discussion regarding the framework and fundamental issues of the model appears in Section 3. Section 4 provides additional results. In Section 4.1 we present a natural comparative notion of revealed bias towards the status quo and provide its implications to the probabilistic dominance model. An application of the model is given in Section 4.2 in which we show how it predicts the well-known endowment effect and provide a simple way of calculating the buying and selling prices. In Section 5 we axiomatize probabilistic dominance relations in an Anscombe–Aumann framework and discuss some properties of these relations. Section 6 suggests additional models where the probabilistic dominance approach could be applied, such as choice with endogenous reference points. Lastly, all the proofs appear in the Appendix.

2. The main model

2.1. The framework.

We follow the setup and notation in Masatlioglu and Ok (2005) with the difference that the objects of choice here are assumed to be Anscombe and Aumann acts. Formally, let X be a non–empty finite set of *prizes*,[10] and let $\Delta(X)$ be the set of all *lotteries* (probability distributions) over X.[11] Given $p \in \Delta(X)$ and $x \in X$, we denote by $p(x)$ the probability p assigns to the prize x. Let S be a finite non–empty set of *states of nature*. Now, consider the collection $\mathcal{F} = \Delta(X)^S$ of all functions from states of nature to lotteries. Such functions are referred to as *acts*. We denote by $\mathcal{F}_c$ the collection of all constant acts. Following the standard abuse of notation, we denote by p

[10] See Remark 1 below.

[11] Given a finite set A, $\Delta(A)$ denotes the collection of all probability distributions over A.

the constant act that assigns the lottery p to every state of nature. Similarly, given an act f and a state s, we write $f(s)$ to represent the constant act that returns the lottery $f(s)$ in every state of nature.

Mixtures (convex combinations) of acts are performed state-wise. For $f, g \in \mathcal{F}$ and $\alpha \in [0,1]$, we denote by $f \oplus_\alpha g$ the act $\alpha f + (1-\alpha) g$ that returns $\alpha f(s) + (1-\alpha) g(s)$, for each state $s \in S$. And, if A is a non-empty collection of acts, we write $A \oplus_\alpha g$ to denote the collection $\{f \oplus_\alpha g \ : \ f \in A\}$. In addition, for any act $f \in \mathcal{F}$, state $s^* \in S$, lottery $p \in \Delta(X)$ and $\lambda \in [0,1]$, we define $f \oplus_\lambda^{s^*} p \in \mathcal{F}$ by $(f \oplus_\lambda^{s^*} p)(s^*) := \lambda f(s^*) + (1-\lambda) p$ and $(f \oplus_\lambda^{s^*} p)(s) := f(s)$ if $s \neq s^*$. Given a collection of acts A, $A \oplus_\lambda^{s^*} p$ denotes the collection $\{f \oplus_\lambda^{s^*} p \ : \ f \in A\}$.

Let $\mathfrak{F}$ denote the set of all nonempty closed subsets of $\mathcal{F}$.[12] The symbol $\diamond$ will be used to denote an object that does not belong to $\mathcal{F}$. By a *choice problem* we mean a list (A, f) where $A \in \mathfrak{F}$ and either $f \in A$ or $f = \diamond$. The set of all choice problems is denoted by $\mathcal{C}(\mathcal{F})$. If $f \in A$, then the choice problem (A, f) is referred to as a *choice problem with a status quo*. The interpretation is that the agent has to make a choice from the set A while the alternative f is her default option. We denote by $\mathcal{C}_{sq}(\mathcal{F})$ the set of all choice problems with a status quo. Finally, the notation $(A, \diamond)$, with $A \in \mathfrak{F}$ is used to represent a choice problem without a status quo.

The decision maker (henceforth, DM) is associated with a *choice correspondence*, that is, a map $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ such that

$$c(A, f) \subseteq A \text{ for all } (A, f) \in \mathcal{C}(\mathcal{F}).^{13}$$

Remark 1. *Although we assume the prize space X to be finite, this is not necessary for the results in the paper. For example, all results are still true, with essentially no*

[12] All results remain true, with no modifications, if we assume $\mathfrak{F}$ to be the set of all non-empty finite subsets of $\mathcal{F}$.

[13] As in some of the papers discussing choice in the presence of status quo, it is also possible to consider a system of preference relations as the primitive. This may lead to unresolved issues though. For example, if a status-quo f is preferred to both g and h, then in the presence of f as a status quo, the ranking between g and h is never revealed. Thus, it seems more natural for the primitive to be a choice correspondence. Apesteguia and Ballester (2009) study the relation between the two possible primitives.

modifications in the proofs, if X is a compact metric space and $\Delta(X)$ is the space of Borel probability measures on X. With minor modifications in the proofs we can also derive all the results for the case where X is a generic set and $\Delta(X)$ is the space of simple lotteries over X. However, in this case we need some adaptation in the definition of $\mathfrak{F}$, since we no longer have a natural topology we can use to define the collection of compact subsets of $\mathcal{F}$. Everything works flawlessly if we define $\mathfrak{F}$ to be the collection of finite subsets of $\mathcal{F}$, though.

2.2. **Representation.** We define a probabilistic dominance choice model as follows:

Definition 1. *A correspondence $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ is a* probabilistic dominance choice correspondence *if there exist an affine function $u : \Delta(X) \to \mathbb{R}$, a prior π over S and a $\theta \in [0,1]$ such that, for all $A \in \mathcal{F}$,*

$$c(A, \diamond) = \arg\max_{f \in A} \int_S u(f(s)) d\pi$$

and, for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A, g) = \arg\max_{f \in \mathcal{D}(A,g,\pi,\theta)} \int_S u(f(s)) d\pi,$$

where, for each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\mathcal{D}(A, g, \pi, \theta) := \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$.

The interpretation of this choice procedure is the following. In the absence of a status quo, the agent acts as a standard subjective expected utility maximizer. As elaborated in the Introduction, when faced with a decision problem governed by a status quo the agent wishes to be confident that, regardless of her choice, she will obtain an outcome at least as good as the one she would have obtained had she retained the status quo, with sufficiently high probability. She first eliminates all those alternatives that do not ensure such a sufficiently high level of confidence. Following the elimination stage, she acts as a standard expected utility maximizer as in choice problems without a status quo.[14]

[14]In a different framework and in a status-quo free context, sequentially rationalizable choices were also studied by Manzini and Mariotti (2007), Cherepanov, Feddersen, and Sandroni (2010) and Apesteguia and Ballester (2010). Two-staged decision processes are also of importance in the marketing literature (see Sheridan, Richards, and Slocum (1975) and Gensch (1987) and references therein). Experimental and empirical evidence point to the fact that, when facing a choice problem, individuals

Two points should be noted regarding the threshold parameter θ. One is that θ captures the degree of confidence the DM wishes to ensure when moving away from the status quo. In other words, θ describes the distinction (discussed in the Introduction) between the status quo and new alternatives. Fixing the beliefs and tastes, it is intuitive that the larger the threshold, the more bias towards the status quo the DM is going to exhibit (this intuition is made formal, even for the case where beliefs are not fixed, in Section 4.1). When $\theta = 0$ the DM exhibits no bias at all towards the status quo and acts as a standard expected utility maximizer. At the other end of the threshold range, a DM who is characterized by $\theta = 1$ is not willing to take any chances of loosing by moving away from the status quo and displays the extreme approach of unanimity. Second, the bias towards the status quo is not continuously shifting as θ changes. This point is closely related to the non-uniqueness of θ in the representation of c and it is discussed in detail in Section 2.4.

Before continuing with the analysis of the suggested model, we would like to point out to its interpretation in a framework of choices of committees, where each "state of the world" is considered to be a voter in a committee. Given such interpretation, each alternative including the status quo (say, the current policy enforced by the company) is associated with an opinion profile. At first the committee votes on a short list—each member votes for the alternatives that he or she perceives better than the status quo, and only alternatives with sufficiently many votes are considered. Next, the committee proceeds with a more thoughtful discussion to decide on the final choice.

2.3. **Axioms.** We impose the following properties on a choice correspondence c.

A1 *WARP.* If $(A,h),(B,h) \in \mathcal{C}(\mathcal{F})$ are such that $B \subseteq A$ and $c(A,h) \cap B \neq \emptyset$, then $c(B,h) = c(A,h) \cap B$.

The first postulate is an adaptation of the standard Weak Axiom of Revealed Preference to the environment here. It says that if we keep the status quo (or the absence of status quo) fixed, then c satisfies that postulate. This axiom implies that, for each

tend to eliminate some alternatives according to a crude, non-compensatory initial rule, followed by a more thoughtful and compensatory process for deciding on the final choice.

$f \in \mathcal{F} \cup \{\diamond\}$, there exists a complete preorder $\succeq_f$ that represents the DM's behavior when the default option is f.

A2 *Independence.* For any $f \in \mathcal{F}$, $A \in \mathfrak{F}$ and $\lambda \in [0,1]$, $c(A \oplus_\lambda f, \diamond) = c(A, \diamond) \oplus_\lambda f$.

Our Independence axiom is also standard. It says that the agent's choices in problems without a status quo satisfy the well-known Independence postulate.

A3 *Continuity.* For any $f, g, h \in \mathcal{F}$, the following sets are closed: $\{\alpha : f \oplus_\alpha g \in c(\{f \oplus_\alpha g, h\}, \diamond)\}$, $\{\alpha : h \in c(\{f \oplus_\alpha g, h\}, \diamond)\}$, $\{\alpha : f \oplus_\alpha g \in c(\{f \oplus_\alpha g, h\}, h)\}$ and $\{\alpha : h \in c(\{f \oplus_\alpha g, h\}, f \oplus_\alpha g)\}$.

The first part of our continuity condition, which talks about choices in problems without a status quo, is entirely standard. The second part imposes a similar continuity condition in problems with a status quo.

A4 *Monotonicity.* If $f(s) \in c(\{f(s), g(s)\}, \diamond)$ for all $s \in S$, then, for any $h \in \mathcal{F}$, $g \in c(\{g, h\}, h)$ implies $f \in c(\{f, h\}, h)$ and $h \in c(\{f, h\}, f)$ implies $h \in c(\{g, h\}, g)$.

This axiom is the standard monotonicity postulate adapted to a model of status quo bias. Assuming that g is chosen in the presence of h, despite h being the status quo, it is natural to assume that the bias towards h will reduce whenever presented with an act f that is unambiguously better than g, in the sense that f performs better than g in all states. So the axiom imposes that such an act f is also chosen in the presence of h when h is the status quo. Similarly, if h is such a good option that it is chosen in the presence of f even when f is the status quo, then it is natural to think that h will also be chosen when confronted with a status quo g that is unambiguously worse than f.

A5 *Status quo Irrelevance (SQI).* For any $(A, f) \in \mathcal{C}_{sq}$, if there does not exist a non-singleton $B \subseteq A$ with $(B, f) \in \mathcal{C}_{sq}$ and $\{f\} = c(B, f)$, then $c(A, f) = c(A, \diamond)$.

The postulate above was first introduced by Masatlioglu and Ok (2010). It begins with a set A such that in no subset of A the bias towards the status quo f is strong enough to make it the only choice. In other words, the bias towards f is not significant, in the sense that the DM always considers moving away from it as something acceptable. The axiom then requires that such a default option does not affect the DM's behavior. That is, she resolves her problems as if there existed no status quo.

There are two forces behind **A5**. First, there is the idea that a weak status quo should not affect the agent's choices. Arguably, the following postulate is a better description of this idea:

A5a *Weak status quo Irrelevance (WSQI).* For any $(A, f) \in \mathcal{C}_{sq}$, if there does not exist a non-singleton $B \subseteq A$ with $(B, f) \in \mathcal{C}_{sq}$ and $f \in c(B, f)$, then $c(A, f) = c(A, \diamond)$.

The other force behind **A5** is the idea that in the presence of a status quo the individual's decision making procedure has two stages and the second stage agrees with her choices when there is no status quo. One consequence of this fact would be the following property:

A5b *Two Stages Decision.* For any $f, g \in \mathcal{F}$, if $\{f, g\} = c(\{f, g\}, f)$, then $\{f, g\} = c(\{f, g\}, \diamond)$.

When c satisfies WARP the two properties above together are equivalent to **A5**.

A6 *Single-state Mixing Irrelevance (SSMI).* For any $s^* \in S$, $\lambda \in (0, 1)$, $p \in \Delta(X)$ and $(A, f) \in \mathcal{C}_{sq}$, if $c(B \oplus_\lambda^{s^*} p, \diamond) = c(B, \diamond) \oplus_\lambda^{s^*} p$ for all $B \subseteq A$, then $c(A \oplus_\lambda^{s^*} p, f \oplus_\lambda^{s^*} p) = c(A, f) \oplus_\lambda^{s^*} p$.

Choosing from a set of alternatives, in a given state the agent may or may not obtain a better outcome than the status quo. Mixing all the alternatives (including the status quo) in this state with some other outcome will not change the relative result in this state (or any other). Such a mixing yields a decision problem which is a very simple variation of the original one: no matter what the DM chooses, if s^* is the realized state, then with probability λ she obtains the lottery p, and with probability $1-\lambda$ she obtains the same outcome she would have obtained if there was no such mixing.

Assume that in a specific problem such a mixture is insignificant when the DM objectively assesses the different alternatives (that is, in the status quo free problem), in the sense that it does not affect her choice (in every subset of feasible alternatives). Then **A6** imposes that in the presence of a status quo, such a mixture will not affect the DM's choice as well. In other words, when choices in problems that are not governed by a status quo are strongly robust to a single-state mixing, then the choices in the presence of a status quo should also be robust to such mixing.

A7 *Binary Consistency.* Let $p, q, r \in \Delta(X)$. If f and g are acts such that $f(S) \cup g(S) \subseteq \{p, r\}$, then $\{f\} = c(\{f, g, q\}, q)$ implies that $\{f\} = c(\{f, g, q\}, \diamond)$.

To describe **A7** in detail, define the relation $\succcurlyeq \subseteq \mathcal{F} \times \mathcal{F}$ by $f' \succcurlyeq g'$ if and only if $f' \in c(\{f', g'\}, \diamond)$ and suppose, without loss of generality, that $p \succcurlyeq r$. It is possible to show that unless $p \succ q \succ r$, Binary Consistency is implied by the axiomatic structure above. However, if indeed $p \succ q \succ r$, f and g can be interpreted as acts that either return a lottery that is better than the status quo lottery q or a lottery that is worse. When we learn that f is the unique choice out of $\{f, g, q\}$ when q is the status quo, we must conclude that the event in which f returns the good lottery is more salient, under some subjective criterion, than the event in which g returns the good lottery. The axiom is then a consistency property that imposes that this saliency is sustained in the status quo free problem.

Remark 2. *The axioms SQI and SSMI are written in a stronger format than what is needed. It is easy to see that in the presence of WARP it is enough to require that those axioms be valid for sets with three or less elements. We note that if we replace A5 and A6 by the simpler versions (only for sets with less than four elements) discussed above, then, except for continuity, our axiomatic system is entirely testable.*

2.4. **The representation theorem.** We now provide the main representation result of the paper.

Theorem 1. *Given a choice correspondence $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ the following are equivalent:*
*(1) c satisfies **A1-A7**;*
(2) c is a probabilistic dominance choice correspondence.

Remark 3. *If c is not trivial,*[15] *then **A1-A7** imply that the prior π is unique and the function u is non-constant and unique up to positive linear transformations. The same is not true for θ – following the proof of Theorem 1, it is clear that θ can be chosen from some interval. For example, consider a state space with two states and a probabilistic dominance choice correspondence that is represented by a non-constant utility and a prior distribution assigning probability* 0.5 *for each state. Holding the utility and prior fixed, one would obtain identical choices for every $0 \leq \theta \leq 0.5$. For each such θ, in every choice problem the corresponding procedure would simply pick those alternatives with highest expected utility.*

Next we present an auxiliary result, which is interesting on its own, partially hints to the proof of Theorem 1, and clarifies the uniqueness issue of the probabilistic threshold parameter θ representing c.

Proposition 1. *Given a choice correspondence $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ the following are equivalent:*
*(1) c satisfies **A1-A6**;*
(2) there exists an affine function $u : \Delta(X) \to \mathbb{R}$, a prior π over S and a non-empty collection of events $\mathcal{T} \subseteq 2^S$ such that, for all $A \in \mathcal{F}$,

$$c(A, \diamond) = \arg\max_{f \in A} \int_S u(f(s))d\pi$$

and, for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A, g) = \arg\max_{f \in \mathcal{D}(A,g,\mathcal{T})} \int_S u(f(s))d\pi,$$

where, for each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\mathcal{D}(A, g, \mathcal{T}) := \{f \in A : u(f(s)) \geq u(g(s))$ for all $s \in T$ for some $T \in \mathcal{T}\}$.
Moreover, if c is non-trivial then π is unique, u is non-constant and unique up to positive linear transformations, and there exists a unique collection of events $\mathcal{T}$ that is closed

[15] A choice correspondence c is trivial if, for all $c(A, g) \in \mathcal{C}(\mathcal{F})$, $c(A, g) = A$.

under containment,[16] *satisfies that* $\pi(T) > 0$ *for all* $T \in \mathcal{T}$ *and represents* c *in the sense above.*

The interpretation of the proposition is similar to that of Theorem 1. The events in the collection $\mathcal{T}$ are called *decisive*. In the presence of a status quo, during the first stage of the decision process, an act has to perform at least as well as the status quo for all states inside at least one of the *decisive events*. Differently from Theorem 1, Proposition 1 imposes no restrictions on the collection of events $\mathcal{T}$. In particular, $\mathcal{T}$ need not be consistent with the agent's subjective belief π and there might exist some decisive event $T \in \mathcal{T}$ and an event $T' \notin \mathcal{T}$ such that $\pi(T) < \pi(T')$. Such behavior can be explained by the intuitive–deliberate choice theory described by Kahneman (2003), but this raises two remarks. First, it is hard to believe that a DM would base his choices on a low probability event but not on an event with higher probability. And second, going through all the difficulties entailed in computing the subjective prior, it seems unnatural that the DM would not revise the first stage of the decision process and reconstruct $\mathcal{T}$ so that it contained all high probability events. In short, the probabilistic dominance choice correspondence characterized in Theorem 1 is a special case of the representation above where $\mathcal{T} := \{T \subseteq S : \pi(T) \geq \theta\}$.

Remark 4. *In Proposition 1, the uniqueness properties of the collection* $\mathcal{T}$ *of decisive events which represents* c *are quite natural. First, given any* $T \in \mathcal{T}$*, if an act* f *dominates the status quo* g *over an event* T' *that contains* T*, then it dominates* g *over* T *itself, thus* f *passes to the second stage of the decision process. Second, consider a* π*-null event* T *and assume that* f *dominates the status quo* g *only over* T*. This implies that the expected utility of* f *is strictly lower than that of* g*, thus* f *will never be chosen when* g *is the status quo. This shows that zero probability events are completely inconsequential for the choice procedure described in Proposition 1, therefore, including or not including them in the collection* $\mathcal{T}$ *does not affect the DM's choices.*[17]

[16] A collection of events $\mathcal{T}$ is closed under containment if $T \in \mathcal{T}$ and $T \subseteq T'$ implies that $T' \in \mathcal{T}$.

[17] Although correct in the current formulation, Proposition 1 leads to a revised definition of $\mathcal{D}(A, g, \pi, \theta)$ in Theorem 1: $\mathcal{D}(A, g, \pi, \theta)$ could be defined by $\{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\} \cap \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} > 0\}$.

3. Discussion

3.1. **Savagean framework.** The current paper adopts the Anscombe and Aumann setup. This is done solely for the sake of tractability. The probabilistic dominance choice model can be easily formulated in a Savagean framework, however, there is a price to pay due to the axiomatic structure in such a setting.

In the case of a finite state space, the Gul (1992) axioms would be postulated instead of the Anscombe–Aumann axioms in order to obtain the expected utility representation for the status-quo free choice problems. Other than that, the main difference with a corresponding Theorem 1 in Savage's framework would be the reformulation of **A6**, since in an Anscombe and Aumann setup we could utilize the convex structure of the domain of acts. This, of course, cannot be done in Savage's framework. Nevertheless, going carefully through the details of the proof of Theorem 1, it is clear that using mixtures is not the essential part of **A6**. The power behind this axiom is its use of ordinal preserving single-state alterations, and that can easily be accommodated.

In the classical Savage (1954) infinite state space, **A6** would have to be written in a somewhat stronger fashion, since a single state no longer has any meaningful "contribution" to the assessment of an act. In this case, the reformulated axiom would take into account ordinal preserving variations over events. A side benefit of the model's formulation in such a framework would be the uniqueness of the threshold parameter θ.

3.2. **Biased attitudes towards lotteries with objective and subjective probabilities.** The probabilistic dominance choice model, as formulated in an Anscombe and Aumann framework, differentiates between biased attitudes with objective and subjective probabilities. In particular, the decision maker in this model exhibits biased behavior only towards lotteries with subjective probabilities. If she is to choose between two acts that yield identical expected utility in every state, regardless of the objective distribution over outcomes in each state, then she will reveal no bias and her choice would be the two acts, no matter if one of them (or neither of them) is the status quo. An important point to note is that this difference in attitudes is a feature of the framework and would not be an issue in the general setup of Savage, in which the uncertainty is only over the realized state and not the outcome.

Nevertheless, it is possible to think about a situation in which it is natural to differentiate between bias attitudes with objective and subjective probabilities. As (one of the interpretations) we have discussed in the Introduction, we have in mind a DM that considers the anticipated (ex-post) regret at the time she makes her decisions. For example, consider a CEO subordinate to a board of directors. The CEO has to take an important strategic decision for the company at present time, but its ramifications depend on the state of nature yet to be realized. If the probabilities governing such a decision are objective, the CEO's choice can easily be justified for the board of directors. Indeed, one would expect that in this case the company would have an explicit decision making policy. On the other hand, if the probabilities are subjective the CEO knows that a bad outcome in the future will most likely be seen as a sign of incompetence. This shows that there is good reason to believe that biased attitudes with objective and subjective probabilities may be very different.

3.3. **Continuity.** One may argue that the model above has unrealistic predictions. In words, an act that is essentially the same as the status quo in most states and much better in some other states can be rejected. For example, consider a world with three states where the decision maker believes that the three states are equally likely. Moreover, suppose $\theta = 1/2$ and let f be a status quo act that returns one dolar in each state. Now consider an act g that returns $1-\varepsilon$ in the first two states and returns M in the last state. Independently of how small ε or how large M are, our model implies that the decision maker chooses f over g when f is the status quo.

The uneasiness of the example above is a consequence of the fact that models of mentally constrained optimization, in particular, and of sequential decision making, in general, induce discontinuous choice correspondences. While most of the sequential choice literature works with a finite set of alternatives and, consequently, such an issue does not make an appearence, here the use of the Ancombe-Aumann framework emphasizes that, since it provides a natural measure of how similar two acts are. Although the example above does not apply to the reformulation of the probabilistic dominance choice model in a Savage (1954) framework, it is possible to generate discontinuities in such a setting as well due to the non-atomicity of the subjective prior.

We view the behavior described by the model introduced here as an approximation and consider the extreme consequences suggested by the example above as a price we have to pay to obtain a tractable model. Nevertheless, decision making procedures very similar to the one described here are used in practice. One example of that is the use of the value at risk measure as a regulatory device for managers of trusts and pension plans. In such situations, managers operate with the additional constraint of having to avoid portfolios that can incur a big loss with a probability above a certain threshold. This procedure is very much in the spirit of our model and exhibits the same type of discontinuities.

4. Additional results

4.1. **Comparative status quo bias.** Consider two DMs, I and II, associated with choice correspondences c_1 and c_2, respectively. Assume that there exists a status quo free choice problem from which both DMs choose f. Now, assume that when a given alternative g is the status quo, f is still chosen by I. If II exhibits less bias towards the status quo than I, then, in the presence of the same status quo, we would expect that she choose f as well. This is the idea behind the following comparative notion.

Definition 2. c_1 reveals more bias towards the status quo *than* c_2 *if, for every* $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$ *such that* $f \in c_1(A, \diamond)$ *and* $f \in c_2(A, \diamond)$,

$$f \in c_1(A, g) \implies f \in c_2(A, g).$$

The following proposition reinforces the intuition behind our interpretation of probabilistic dominance choice and in particular, the formation of decisive events.

Proposition 2. *Suppose* c_1 *and* c_2 *have representations as in Proposition 1, with the same utility function* u*, priors* π_1 *and* π_2*, and collections of decisive events* $\mathcal{T}_1$ *and* $\mathcal{T}_2$*, respectively. Then, the following are equivalent:*
(1) c_1 *reveals more bias towards the status quo than* c_2*;*
(2) the tuple $(u, \pi_2, \mathcal{T}_1 \cup \mathcal{T}_2)$ *also represents* c_2 *in the sense of Proposition 1.*

What the proposition suggests is simply that, revealing less status quo bias is equivalent to imposing less restrictions on the considered alternatives by having a richer collection of decisive events (up to null events). In the context of Theorem 1, if the DMs share utilities and beliefs then Proposition 2 implies that, whenever $c_1 \neq c_2$, the paramenter θ_1 used in the representation of c_1 is strictly greater than the parameter θ_2 used in the representation of c_2. This is formally presented in the next proposition.

Proposition 3. *Suppose c_1 and c_2 have representations as in Theorem 1, with the same utility function u and prior π, and threshold parameters θ_1 and θ_2, respectively. Then, the following are equivalent:*
(1) c_1 reveals more bias towards the status quo than c_2;
(2) $c_1 = c_2$ or $\theta_1 > \theta_2$.

4.2. **The endowment effect.** People often demand more in order to give up a commodity than they would be willing to pay to acquire it (see Kahneman, Knetsch, and Thaler (1991) for a survey). This phenomenon, intrinsically related to status quo bias is the well-documented *endowment effect* (Thaler (1980)). We show that our model predicts this phenomenon in terms of unambiguous gains of utility. Also, we provide a simple way to compute, in terms of utility, the willingness to accept (WTA) and willingness to pay (WTP).

Consider a probabilistic dominance choice correspondence c that has a representation with a non-constant and affine function $u : \Delta(X) \to \mathbb{R}$, a prior π over S and a threshold parameter $\theta \in [0, 1]$. We define the function $S_c : \mathcal{F} \to u(\Delta(X))$ by[18]

$$S_c(f) := \inf\{u(p) : p \in \Delta(X) \text{ and } p \in c(\{p, f\}, f)\}$$

and the function $B_c : \mathcal{F} \to u(\Delta(X))$ by

$$B_c(f) := \sup\{u(p) : p \in \Delta(X) \text{ and } f \in c(\{p, f\}, p)\}.$$

Intuitively, $S_c(f)$ is the minimum unambiguous gain the individual would require in order to give away the act f (WTA in terms of utility) and $B_c(f)$ is the maximum unambiguous gain the individual would be willing to give away in order to have f (WTP in terms of utility).

For a constant act p to be chosen from the pair $\{f, p\}$ when f is the status quo, two conditions must be fulfilled. It must dominate f with probability at least θ, and its utility

[18]Similar definitions, in an environment with no uncertainty and explicit potential prices, can be found in Masatlioglu and Ok (2005).

must be as high as the expected utility induced by f. Thus, given the representation of c, we have that

$$S_c(f) = \max\left\{\int_S u(f(s))d\pi, \min\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \leq v\} \geq \theta\}\right\}. \tag{1}$$

Applying similar arguments as above (for the case where the constant act p is the status quo when choosing between f and p), we obtain

$$B_c(f) = \min\left\{\int_S u(f(s))d\pi, \max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}\right\}. \tag{2}$$

Loosely speaking, the two terms above, $\min\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \leq v\} \geq \theta\}$ and $\max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}$, are expressions for the left θ quantile and the right $1-\theta$ quantile of the distribution of $u(f)$ with respect to π. We denote them $\widetilde{Q}_{\pi,\theta}(f)$ and $\widehat{Q}_{\pi,1-\theta}(f)$ respectively. Thus, the WTA and WTP of any act can be calculated using only its expected value and these two quantiles.

When c can be represented with $\theta = 0$ and the DM is not status quo biased, choosing as a standard expected utility maximizer, for every act f we have that $\widehat{Q}_{\pi,1}(f) \geq \int_S u(f(s))d\pi \geq \widetilde{Q}_{\pi,0}(f)$, meaning that $B_c(f) = S_c(f)$. The question is how $B_c(f)$ and $S_c(f)$ relate whenever c reveals a bias towards the status quo and cannot be represented with $\theta = 0$.

From Eq. (1) and (2) it is easy to verify that, for a particular act f, the WTA is at least as high as the WTP. Also, a necessary and sufficient condition for both to be equal is $\widehat{Q}_{\pi,1-\theta}(f) \geq \int_S u(f(s))d\pi \geq \widetilde{Q}_{\pi,\theta}(f)$. Furthermore, if c cannot be represented with $\theta = 0$, there always exists an act f for which the latter inequalities do not hold. This implies that the WTA for this specific act is strictly higher than the WTP. We formalize these results in the next proposition:

Proposition 4. *Suppose that u, π and θ represent a non-trivial probabilistic dominance choice correspondence c as in Theorem 1. Then,*
(a) $S_c(f) \geq B_c(f)$ for every $f \in \mathcal{F}$.
(b) For every $f \in \mathcal{F}$, $S_c(f) = B_c(f)$ if and only if $\widehat{Q}_{\pi,1-\theta}(f) \geq \int_S u(f(s))d\pi \geq \widetilde{Q}_{\pi,\theta}(f)$.
(c) The following statements are equivalent:
   (1) There exists $f \in \mathcal{F}$ with $S_c(f) > B_c(f)$;
   (2) There exists $s^* \in S$ such that $\theta > \pi(s^*) > 0$; and
   (3) $\theta = 0$ does not represent c.

5. Probabilistic dominance

Given a representation as in Theorem 1, define the relation $\succeq$ by

$$f \succeq g \iff \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta.$$

We say that $\succeq$ has a *probabilistic dominance* representation and refer to $\succeq$ as a probabilistic dominance relation. As mentioned in the Introduction, $\succeq$ can be interpreted as representing the fact that the probability that the agent would regret choosing f instead of g is below the threshold $1-\theta$.

Being the first of the two steps constructing the choice procedure described in Theorem 1, we discuss in detail this family of relations in the current section.

5.1. **Framework and axioms.** Again, let S be a finite set of states of the world, X be a finite set of alternatives, $\Delta(X)$ be the set of lotteries on X and $\mathcal{F} := \Delta(X)^S$ be the space of Anscombe and Aumman acts. The primitive of the model now is a binary relation $\succeq \subseteq \mathcal{F} \times \mathcal{F}$. For any $f, g \in \mathcal{F}$ and event $T \subseteq S$ we write $f \trianglerighteq_T g$ to represent the fact that $f(s) \succeq g(s)$ for all $s \in T$.

The following is a list of basic assumptions (axioms) about $\succeq$:

B1 *Relation.* $\succeq$ is complete over $\mathcal{F}_c$, reflexive and non-trivial.[19]

B2 *Unambiguous Transitivity.* (i) $f \succeq g$ and $g \trianglerighteq_S h$ imply $f \succeq h$; and (ii) $f \succeq g$ and $h \trianglerighteq_S f$ imply $h \succeq g$.

B3 *Continuity.* For any $f, g, h \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succeq h\}$ and $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \preceq h\}$ are closed.

We concentrate on deviations from standard rationality that are ultimately due to uncertainty. Therefore, we assume that the restriction of $\succeq$ to constant acts satisfies all the standard postulates of rationality. Specifically, **B1** imposes that the restriction of $\succeq$ to constant acts is complete. This restriction is transitive thanks to **B2** and satisfies continuity due to **B3**. Lastly, the restriction of $\succeq$ to constant acts satisfies the standard independence axiom thanks to **B4** that will be presented below.

[19]By non-trivial we mean that $\succeq \neq \mathcal{F} \times \mathcal{F}$.

In the presence of uncertainty, we allow for the possibility that the DM is undecided and we do not require her decisions to be transitive. That is, we do not ask $\succeq$ to be complete nor transitive in general. Despite that, assume that the DM is able to conclude that $f \succeq g$. On top of that, assume that the act h is dominated by g (or dominates f) in the sense that it assigns to every state $s \in S$ a lottery that is weakly worse (better) than the lottery returned by g (f). So, in some sense, h is worse (better) than g (f) independent of any uncertainty related considerations. **B2** requires that in such a situation the DM considers $f \succeq h$ ($h \succeq g$). Note that **B2**, along with the reflexivity of $\succeq$, implies that $\succeq$ satisfies the standard monotonicity axiom. That is, for any two acts f and g, if $f \trianglerighteq_S g$, then $f \succeq g$. Also, as we have discussed above, **B2** implies that $\succeq$ is transitive over $\mathcal{F}_c$.

B3 is a standard technical continuity condition of preferences under uncertainty.

Consider any two acts f and g. Fix a state $s^* \in S$, a lottery $p \in \Delta(X)$ and $\lambda \in (0, 1)$. If the DM's choices over constant acts satisfy the standard independence axiom, then $\lambda f(s^*) + (1 - \lambda)p \succeq \lambda g(s^*) + (1 - \lambda)p$ if and only if $f(s^*) \succeq g(s^*)$. So, mixing f and g in a single state with the same lottery does not change the states where f performs better than g or g performs better than f. We can then conclude that the DM's decision about any two acts f and g has to agree with her decision about $f \oplus_\lambda^{s^*} p$ and $g \oplus_\lambda^{s^*} p$, for any $s^* \in S$, $p \in \Delta(X)$ and $\lambda \in (0, 1)$. This is exactly what the postulate below imposes.[20]

B4 *State-wise Independence.* For any two acts f and g, lottery $p \in \Delta(X)$, state $s^* \in S$ and $\lambda \in (0, 1)$, $f \succeq g$ if and only if $f \oplus_\lambda^{s^*} p \succeq g \oplus_\lambda^{s^*} p$.

We note that a simple inductive argument shows that **B4** is stronger than the standard Independence axiom, **B4',** below.[21]

[20]Behind this discussion there is the assumption that no intensity considerations about how better or how worse a given act performs in each state are relevant for the DM's decisions. There is empirical evidence that the beliefs of obtaining a worse outcome than the alternative is the dominant aspect (out of the two mentioned above) that affects the agent's choice when anticipated regret is in play (e.g., Ritov (1996) and references within). Of course, completely disregarding the intensity of the anticipated regret is a bit extreme. We consider it as a useful simplification that delivers a tractable model.

[21]This is also clear from claim 1 in the proof of Proposition 5, in the Appendix.

B4' *Independence.* For any acts $f, g, h \in \mathcal{F}$ and $\lambda \in (0,1)$, $f \succeq g$ if and only if $\lambda f + (1-\lambda)h \succeq \lambda g + (1-\lambda)h$.

Similarly to the result in the previous section, **B1–B4** yield the following useful preliminary result:

Proposition 5. *Given a binary relation $\succeq$ over $\mathcal{F}$, the following are equivalent:*
*(1) $\succeq$ satisfies **B1–B4**;*
(2) there exist a non–constant and affine function $u : \Delta(X) \to \mathbb{R}$ and a non–empty collection of events $\mathcal{T} \subseteq \mathbf{2}^S \setminus \{\emptyset\}$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ if and only if } u(f(s)) \geq u(g(s)) \text{ for all } s \in T, \text{ for some } T \in \mathcal{T}. \tag{3}$$

Moreover, u is unique up to positive linear transformations and there exists a unique collection of events $\mathcal{T}$ that is closed under containment and represents $\succeq$ in the sense above.

Call an event $T \subseteq S$ *decisive* if $f(s) \succeq g(s)$ for all $s \in T$ implies that $f \succeq g$. Proposition 5 states, informally, that the axioms discussed above are equivalent to the existence of a collection of decisive events that completely characterizes $\succeq$.

5.2. **Complementarities.** Consider the following example:

Example 1. *Suppose $S = \{s_1, s_2, s_3, s_4\}$, $p \succ r$, and the DM's preferences satisfy axioms **B1–B4** with $\{s_1, s_3\}, \{s_2, s_4\}$ as decisive events. That is, for every two acts f and g, $f \succeq g$ if, and only if, $\{s_1, s_3\} \subseteq \{s : f(s) \succeq g(s)\}$ or $\{s_2, s_4\} \subseteq \{s : f(s) \succeq g(s)\}$. Now, let $f := (p, r, p, r)$, $f' := (r, p, r, p)$, $g := (p, p, r, r)$ and $g' := (r, r, p, p)$. We have that $\frac{1}{2}g + \frac{1}{2}g' \trianglerighteq_S \frac{1}{2}f + \frac{1}{2}f'$, $f \succeq p$ and $f' \succeq p$, but neither $g \succeq p$ nor $g' \succeq p$ are true.*

In Example 1, we see that the decision rule represented by $\succeq$ exhibits a form of perfect complementarity between the states s_1 and s_3, and the states s_2 and s_4. That is, for the DM the fact that an act f returns a better outcome than another act g in state s_1 is only relevant if it is also true that f returns a better outcome than g in state s_3. The same thing occurs with states s_2 and s_4. The example shows that such complementarities yield the following result: even though both f and f' are preferred to p, and even though both g and g' are not preferred to p, the mixture of g's unambiguously dominates the (similar) mixture of f's. The following axiom states that such complementarities between states should not exist.

B5 *No State–Complementarities.* For any $p, q, r \in \Delta(X)$ and finite sequences of acts, $f_1, ..., f_m$ and $g_1, ...g_m$, such that, for all $i = 1, ..., m$, $f_i(S) \cup g_i(S) \subseteq \{p, r\}$, if $\sum_{i=1}^m \lambda_i g_i \trianglerighteq_S \sum_{i=1}^m \lambda_i f_i$ for some $\lambda \in \Delta(m)$ and $f_i \succeq q$ for all i, then $g_i \succeq q$ for some i.

We can now prove the following result:

Theorem 2. *Given a binary relation $\succeq$ over $\mathcal{F}$, the following are equivalent:*
(1) $\succeq$ satisfies ***B1–B5****;*
(2) there exist a non–constant and affine utility function $u : \Delta(X) \to \mathbb{R}$, a probability distribution π on S and a $\theta \in (0, 1]$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \quad \text{if and only if} \quad \pi\big(\{s \in S : \ u(f(s)) \geq u(g(s))\}\big) \geq \theta. \tag{4}$$

Of course, the representation in Theorem 2 is a particular case of the representation obtained in Proposition 5. In the representation above, the collection of decisive events is given by

$$\mathcal{T} := \{T \subseteq S : \pi(T) \geq \theta\}. \tag{5}$$

Example 1 shows that given a collection $\mathcal{T}$ of decisive events, **B5** is necessary for the existence of a probability measure and a threshold parameter that yield $\mathcal{T}$ as described in Eq. 5. Note that in Example 1 the events $\{s_1, s_3\}$ and $\{s_2, s_4\}$ are decisive. So, if $\succeq$ could be represented as in the statement of Theorem 2 we would have $\pi(\{s_1, s_3\}) \geq \theta$ and $\pi(\{s_2, s_4\}) \geq \theta$. It is not hard to see that this would imply that at least one of the events in the collection $\{\{s_1, s_2\}, \{s_1, s_4\}, \{s_3, s_2\}, \{s_3, s_4\}\}$ would also have probability greater than or equal to θ and, therefore, would also be decisive. This contradicts the assumptions within the example.

5.3. **Additional properties and particular cases.** In this section we very briefly discuss some properties and special cases of the representation introduced in Theorem 2.

Uniqueness. Following the proof of Theorem 2, it is clear that there is no unique couple (π, θ) that represents $\succeq$. Consider the following example.

Example 2. *Let $S = \{s_1, s_2, s_3\}$, $\pi(s) = \frac{1}{3}$ for every $s \in S$, and $\theta = \frac{2}{3}$. According to Theorem 2, an act f is (weakly) preferred to an act g if and only if f (weakly) dominates g in at least two out of the three states. However, this is also true for every $\theta \in (\frac{1}{3}, \frac{2}{3}]$ (given that the distribution stated above is fixed). It is possible to construct a different distribution that along with an appropriate θ would represent the same preferences. For example, define π' by $\pi'(s_1) = \frac{1}{6}$, $\pi'(s_2) = \frac{1}{3} + \varepsilon$ and $\pi'(s_3) = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is small enough. Now, given such $\varepsilon > 0$, for every $\theta' \in (\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, the couple (π', θ') represents the same relation as (π, θ).*

For a relation $\succeq$ satisfying the axioms discussed above, denote by $\mathcal{H}_{\succeq}$ the collection of elements $(\pi, \theta) \in \Delta(S) \times (0, 1]$ such that (π, θ) represents $\succeq$. A property of the collection $\mathcal{H}_{\succeq}$ is convexity. That is, whenever both (π, θ) and (π', θ') are elements of $\mathcal{H}_{\succeq}$, then $(\alpha\pi + (1 - \alpha)\pi', \alpha\theta + (1 - \alpha)\theta')$ is also an element of $\mathcal{H}_{\succeq}$, for every $\alpha \in (0, 1)$. It can be shown that any pair $(\pi, \theta), (\pi', \theta') \in \mathcal{H}_{\succeq}$ has to induce exactly the same collection of decisive events. This is formalized in the following result:

Proposition 6. *(π, θ) and (π', θ') represent the same relation $\succeq$ if, and only if, $\{T \subseteq S : \ \pi(T) \geq \theta\} = \{T \subseteq S : \ \pi'(T) \geq \theta'\}$.*

Completeness. The relations we have been working with so far are both incomplete and intransitive, in general. We now characterize a complete binary relation that satisfies **B1–B5**.

It is clear that whenever $\theta \leq 1/2$, the pair $(\pi, \theta) \in \Delta(S) \times (0, 1]$ induces a relation $\succeq$ that is complete. It is not hard to see that the converse is true in the sense of the proposition below.

Proposition 7. *Suppose that $\succeq$ is complete and satisfies **B1–B5**. Then, $(\pi, \theta) \in \mathcal{H}_{\succeq}$ implies that $(\pi, \min\{1/2, \theta\}) \in \mathcal{H}_{\succeq}$.*

Thus, whenever $\succeq$ is complete we can represent it with some $\theta \leq 1/2$.

Transitivity and Bewley's Knightian preferences. Above we studied the characterization of complete probabilistic dominance relations. In this subsection we focus on transitivity. We show that in this case one obtains a particular form of Bewley's Knightian preferences (see Bewley (2002)). Informally, transitivity is equivalent to the existence of an event $T \subseteq S$ such that, f is (weakly) preferred to g if and only if f (weakly) dominates g over the event T. Also, every $\pi \in \Delta(S)$ such that $support(\pi) = T$ and $\theta = 1$ would represent such relations, in the sense of Eq. 4. It turns out that given transitivity, **B5** is redundant. We formalize this observation in the next proposition.

Proposition 8. *Consider a binary relation $\succeq$. The following are equivalent:*
*(1) $\succeq$ satisfies transitivity, **B1-B4** ;*
(2) there exist a non–constant and affine utility function $u : \Delta(X) \to \mathbb{R}$ and a probability distribution π over S such that, for every $f, g \in \mathcal{F}$

$$f \succeq g \ \ \textit{if and only if} \ \ \pi\big(\{s \in S : \ u(f(s)) \geq u(g(s))\}\big) = 1.$$

(3) there exist a non–constant and affine utility function $u : \Delta(X) \to \mathbb{R}$ and an event $\emptyset \neq T \subseteq S$ such that for every $f, g \in \mathcal{F}$

$$f \succeq g \ \ \textit{if and only if} \ \ u(f(s)) \geq u(g(s)) \ \textit{for every} \ \ s \in T.$$

6. Additional models of probabilistic dominance

As discussed in the Introduction, probabilistic dominance can be applied to other choice models. We discuss some possible directions in this section.

6.1. **A general model of probabilistic dominance.** The representation in Theorem 2 is a particular case of a general class of representations. We say that a relation $\succeq$ has a *second–order probabilistic dominance representation* if there exist a non–constant and affine function $u : \Delta(X) \to \mathbb{R}$, a probability distribution μ over $\Delta(S)$ and a $\theta \in (0, 1]$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ iff } \mu\left(\pi \in \Delta(S) : \int u(f(s))\, d\pi \geq \int u(g(s))\, d\pi\right) \geq \theta. \tag{6}$$

The representation above reduces to the one in Theorem 2 whenever the support of the measure μ includes only degenerate priors.

A second-order probabilistic dominance representation is a particular case of Lehrer and Teper (2011)'s justifiable preferences. They introduce the notion of justifiability to axiomatic decision theory and axiomatize Knightian preferences that adhere to justifications and incorporate multiple–multiple priors. They characterize a binary relation $\succeq$ over acts, such that there exist a vN–M utility function u and a collection of closed and convex sets of probability distributions $\mathcal{P}$ over the state space, where $f \succeq g$ if and only if there exists $P \in \mathcal{P}$ such that, with respect to every $p \in P$, the expected value of $u(f)$ is at least as high as that of $u(g)$. Formally, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ if and only if } \max_{P \in \mathcal{P}} \min_{\pi \in P} \{\pi \cdot (u(f) - u(g))\} \geq 0.$$

Such preferences are characterized by **B1–B3** and **B4'**,

Given a relation as in Theorem 2 and denoting by $\mathcal{T}$ the collection of decisive events, the collection $\mathcal{P}$ of sets of probability distributions over S given by $\{conv\{\mathbb{1}_s\}_{s \in T} : \ T \in \mathcal{T}\}$, is a justifiable preferences representation of the same relation. In particular, the characterization of justifiable preferences is the second-order probabilities version of Proposition 5.

Second-order probabilistic dominance can be incorporated into a complete decision process to obtain a more general result than Theorem 1. For example, a DM can be associated with a utility function u, a prior $\pi \in \Delta(S)$, a prior μ over $\Delta(S)$ and a $\theta \in [0, 1]$ such that, for all $A \in \mathcal{F}$,

$$c(A, \diamond) = \arg\max_{f \in A} \int_S u(f(s)) d\pi$$

and, for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A, g) = \arg\max_{f \in \mathcal{D}(A,g,\mu,\theta)} \int_S u(f(s)) d\pi$$

where, for each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\mathcal{D}(A, g, \mu, \theta) := \{f \in A : \mu\{\pi' : \int_S u(f(s)) d\pi' \geq \int_S u(g(s)) d\pi'\} \geq \theta\}$.

The interpretation is similar to that of Theorem 1. In the first stage of the decision process the DM employs second–order probabilistic dominance considerations and, out of the alternatives that are not eliminated, in the second stage she chooses an act that maximizes expected utility.[22]

[22]The second stage of the decision process can be any general preferences. For example, smooth ambiguity preferences (Klibanoff, Marinacci, and Mukerji (2005)) with respect to μ and some increasing real function φ.

We were not able (neither for the binary relation model nor the choice model) to find a condition that guarantees that the sets of multiple-multiple priors $\mathcal{P}$ in the justifiable preferences representation could be generated by a probability measure μ over $\Delta(S)$ and a threshold parameter $\theta \in (0,1]$, as in Eq. 6. This is left for future study.

6.2. **Unfeasible status quo.** In some situations, the status quo is not a feasible alternative. The probabilistic dominance approach could still be applied for such problems. Consider the following choice procedure:

$$c(A, \diamond) = \underset{f \in A}{\arg\max} \int_S u(f(s)) d\pi,$$

and for all (A, g) where g need not be an element of A,

$$c(A, g) = \underset{f \in \mathcal{D}(A,g,\pi,\theta)}{\arg\max} \int_S u(f(s)) d\pi,$$

where $\mathcal{D}(A, g, \pi, \theta) = \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$ if $\{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$ is not empty, and $\mathcal{D}(A, g, \pi, \theta) = A$ otherwise.

Similar to the representation in Theorem 1, the DM considers only alternatives that dominate the status quo with high enough probability. Since the status quo need not be feasible, there may not be such an alternative. In this case the DM chooses according to expected utility maximization.

6.3. **Endogenous references.** Probabilistic dominance considerations can also be applied for choice problems where the reference is not observable, but can rather be extracted from behavior. In such a framework, choice problems would be described only by a collection of feasible alternatives A, and the choice c can be described by the existence of an endogenous reference g such that

$$c(A) = \underset{f \in \mathcal{D}(A,g,\pi,\theta)}{\arg\max} \int_S u(f(s)) d\pi,$$

where $\mathcal{D}(A, g, \pi, \theta) = \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$.

The interpretation is similar to the one in the previous subsection, with the difference that the reference is extracted endogenously from the DM's behavior.

Appendix A. Proofs

A.1. **Proof of Proposition 1.** It is routine to show that the representation implies the axioms, so we only show that the axioms are sufficient for the representation. Define the relation $\succcurlyeq \subseteq \mathcal{F} \times \mathcal{F}$ by $f \succcurlyeq g$ iff $f \in c(\{f,g\},\diamond)$. By WARP, $\succcurlyeq$ is a complete preorder and, for any $A \in \mathfrak{F}$, $c(A,\diamond) = \arg\max(A,\succcurlyeq)$.[23] By Independence, $\succcurlyeq$ satisfies the standard Independence axiom. Continuity implies that $\succcurlyeq$ is continuous.[24] Now suppose that $f(s) \succcurlyeq g(s)$ for all $s \in S$. Monotonicity implies that $f \in c(\{f,g\},g)$. By SQI, $f \in c(\{f,g\},\diamond)$, which is equivalent to saying that $f \succcurlyeq g$. That is, $\succcurlyeq$ satisfies monotonicity. We have just shown that $\succcurlyeq$ satisfies all conditions for an Anscombe and Aumann representation. So, there exists an affine function $u : \Delta(X) \to \mathbb{R}$ and a prior π over S such that, for any $A \in \mathfrak{F}$,

$$c(A,\diamond) = \arg\max_{f\in A} \int_S u(f(s))d\pi. \tag{7}$$

Now define the relation $\succcurlyeq^* \subseteq \mathcal{F} \times \mathcal{F}$ by $f \succcurlyeq^* g$ iff $f \in c(\{f,g\},g)$. We note that $\succcurlyeq^*$ is a reflexive binary relation. For each $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$, define $\mathcal{D}((A,g),\succcurlyeq^*) := \{f \in A : f \succcurlyeq^* g\}$. We need the following claim:

Claim 1. *For every $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$,*

$$c(A,g) = \arg\max_{f\in\mathcal{D}((A,g),\succcurlyeq^*)} \int_S u(f(s))d\pi.$$

Proof of Claim. By WARP, $f \in c(A,g)$ implies that $f \in c(\{f,g\},g)$. So, $c(A,g) \subseteq \mathcal{D}((A,g),\succcurlyeq^*)$. Now pick any $(f,h) \in c(A,g)\times\mathcal{D}((A,g),\succcurlyeq^*)$. By WARP, $f \in c(\{f,g,h\},g)$, and, by SQI, $c(\{f,g,h\},g) = c(\{f,g,h\},\diamond)$. Now (7) implies that $\int_S u(f(s))d\pi \geq \int_S u(h(s))d\pi$. We conclude that $c(A,g) \subseteq \arg\max_{f\in\mathcal{D}((A,g),\succcurlyeq^*)} \int_S u(f(s))d\pi$. In particular, this shows that $\emptyset \neq \arg\max_{f\in\mathcal{D}((A,g),\succcurlyeq^*)} \int_S u(f(s))d\pi$. Now pick $h \in \arg\max_{f\in\mathcal{D}((A,g),\succcurlyeq^*)} \int_S u(f(s))d\pi$ and $f \in c(A,g)$. By SQI we know that $c(\{f,g,h\},g) = c(\{f,g,h\},\diamond)$, which, by (7), implies that $h \in c(\{f,g,h\},g)$. But, by WARP, $c(\{f,g,h\},g) = c(A,g)\cap\{f,g,h\}$. We conclude that $\arg\max_{f\in\mathcal{D}((A,g),\succcurlyeq^*)} \int_S u(f(s))d\pi \subseteq c(A)$. ‖

[23]**Notation:** By $\arg\max(A,\succcurlyeq)$ we mean the set $\{f \in A : f \succcurlyeq g \text{ for all } g \in A\}$.

[24]To be precise, $\succcurlyeq$ satisfies the following property: for every $f,g,h \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succcurlyeq h\}$ and $\{\alpha \in [0,1] : h \succcurlyeq \alpha f + (1-\alpha)g\}$ are closed. This property is sometimes called Archimedean Continuity.

Fix any $\lambda : S \to [0,1]$. For any acts f and g define $f \oplus_{\lambda(.)} g$ to be the act such that $\left(f \oplus_{\lambda(.)} g\right)(s) = \lambda(s) f(s) + (1-\lambda(s))g(s)$ for each $s \in S$. We can prove the following claim:

Claim 2. *Suppose $f \succ g$ and $f \oplus_{\lambda(.)} h \succ g \oplus_{\lambda(.)} h$ for some λ such that $\lambda(s) \in (0,1]$ for all $s \in S$. Then, $f \succcurlyeq^* g$ if and only if $f \oplus_{\lambda(.)} h \succcurlyeq^* g \oplus_{\lambda(.)} h$.*

Proof of Claim. Let $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. Order the states in S in a way that the states not in T come first. That is, write S as $S := \{s_1, s_2, ..., s_{|S|}\}$ with $s_i \notin T$ for $i = 1, ..., (|S| - |T|)$. For each $i = 1, ..., |S|$, let $\lambda^i : S \to (0,1]$ be defined by $\lambda^i(s) := \lambda(s)$ if $s \leq i$ and $\lambda^i(s) := 1$ for $s > i$. The representation of $c(\cdot, \diamond)$ implies that $\{f \oplus_{\lambda^i(.)} h\} = c(\{f \oplus_{\lambda^i(.)} h, g \oplus_{\lambda^i(.)} h\}, \diamond)$ for all i. But then, Statewise Independence implies that $f \succcurlyeq^* g$ iff $f \oplus_{\lambda^1(.)} h \succcurlyeq^* g \oplus_{\lambda^1(.)} h$ iff ... iff $f \oplus_{\lambda(.)} h \succcurlyeq^* g \oplus_{\lambda(.)} h$. $\|$

Without loss of generality, we may assume that $u(\Delta(X)) = [0,1]$. Let $\overline{p}$ and $\underline{p}$ be such that $u(\overline{p}) = 1$ and $u(\underline{p}) = 0$. Call an event T a candidate for decisiveness if,

$$\overline{p} T \underline{p} \succcurlyeq^* \lambda \overline{p} + (1-\lambda)\underline{p} \text{ for some } \lambda \in (0,1). \tag{8}$$

Let $\mathcal{T}$ be the collection of all such events. We can now prove the following claim:

Claim 3. *If $f \succcurlyeq g$, then $f \succcurlyeq^* g$ if and only if $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.*

Proof of Claim. Let $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. Suppose first that f and g are such that $u(f(s)) \neq u(g(s))$ for all $s \in S$ and $f \succ g$. Since $f \succ g$, it must be the case that $\pi(T) > 0$. By the representation of $\succcurlyeq$, this implies that if $\lambda^* \in (0,1)$ is small enough then $\overline{f} := \overline{p} T \underline{p} \succ \lambda^* \overline{p} + (1-\lambda^*)\underline{p} =: \underline{g}$. Let $h \in \mathcal{F}$ be any act such that $\lambda^* < u(h(s)) < 1$ for all $s \in T$ and $0 < u(h(s)) < \lambda^*$ for all $s \in S \setminus T$. Now define $f^\alpha := \alpha f + (1-\alpha)h$ and $g^\alpha := \alpha g + (1-\alpha)h$ for some $\alpha \in (0,1)$ small enough so that, for every $s \in T$, $\lambda^* < u(g^\alpha(s)) < u(f^\alpha(s)) < 1$ and, for every $s \in S \setminus T$, $0 < u(f^\alpha(s)) < u(g^\alpha(s)) < \lambda^*$. By the representation of $\succcurlyeq$, it is clear that $f^\alpha \succ g^\alpha$, so, by the previous claim, $f \succcurlyeq^* g \iff f^\alpha \succcurlyeq^* g^\alpha$. Finally, let $\lambda : S \to (0,1)$ and $j \in \mathcal{F}$ be such that, for every $s \in T$, $u(f^\alpha(s)) = \lambda(s) + (1-\lambda(s))u(j(s))$ and $u(g^\alpha(s)) = \lambda(s)\lambda^* + (1-\lambda(s))u(j(s))$ and, for every $s \in S \setminus T$, $u(f^\alpha(s)) = (1-\lambda(s))u(j(s))$ and

$u(g^\alpha(s)) = \lambda(s)\lambda^* + (1-\lambda(s))u(j(s))$.[25] Two applications of Monotonicity give us that $f^\alpha \succsim^* g^\alpha \iff (\overline{p}T\underline{p}) \oplus_{\lambda(.)} j \succsim^* (\lambda^*\overline{p} + (1-\lambda^*)\underline{p}) \oplus_{\lambda(.)} j$, but, by the previous claim, $(\overline{p}T\underline{p}) \oplus_{\lambda(.)} j \succsim^* (\lambda^*\overline{p} + (1-\lambda^*)\underline{p}) \oplus_{\lambda(.)} j \iff \overline{p}T\underline{p} \succsim^* \lambda^*\overline{p} + (1-\lambda^*)\underline{p}$. Suppose now that it is not true that $u(f(s)) \neq u(g(s))$ for all $s \in S$ and $f \succ g$. Define, for each $\gamma \in (0,1)$, $f^\gamma := \gamma\overline{p} + (1-\gamma)f$ and $g^\gamma := \gamma\underline{p} + (1-\gamma)g$. Let $\gamma^* \in (0,1)$ be small enough so that $\{s \in S : u(f^{\gamma^*}(s)) \geq u(g^{\gamma^*}(s))\} = T$. It is clear that $f^\gamma \succ g^{\hat{\gamma}}$, $u(f^\gamma(s)) \neq u(g^{\hat{\gamma}}(s))$ and $\{s \in S : u(f^\gamma(s)) \geq u(g^{\hat{\gamma}}(s))\} = T$ for all $\hat{\gamma} \in (0,\gamma^*]$ and $\gamma \in (0,\hat{\gamma}]$. If $f \succsim^* g$, two applications of Monotonicity give us that $f^{\gamma^*} \succsim g^{\gamma^*}$ and, by what we have just proved, $T \in \mathcal{T}$. Conversely, if $T \in \mathcal{T}$, again by what we have just proved, $f^\gamma \succsim^* g^{\hat{\gamma}}$ for all $\hat{\gamma} \in (0,\gamma^*]$ and $\gamma \in (0,\hat{\gamma}]$. But then continuity implies that, for all $\hat{\gamma} \in (0,\gamma^*]$, we have $f \succsim^* g^{\hat{\gamma}}$. But now another application of continuity gives that $f \succsim^* g$. $\|$

Combining Claims 1 and 3 above we obtain the desired representation.

Now, assuming non-triviality, if there exists $A \in \mathfrak{F}$ such that $c(A,\diamond) \subset A$, then the uniqueness of a non-constant u and π representing c is immediate. If there exists $(A,f) \in \mathcal{C}_{sq}(\mathcal{F})$ such that $c(A,f) \subset A$, then u is non-constant which in turn implies its uniqueness and π's uniqueness. As for $\mathcal{T}$, Eq. 8 implies that it is closed under containment and does not contain π-null events. Moreover, Eq. 8 implies that non-null events cannot be omitted or included to $\mathcal{T}$, thus it is the unique collection, which is closed under containment and contains no π-null events, that represents c. $\square$

A.2. **Proof of Theorem 1.** Again, the arguments that show that the representation implies the axioms are routine, so we only show that the axioms are sufficient for the representation. By Proposition 1, we know that c can be represented in that fashion for some affine $u : \Delta(X) \to \mathbb{R}$, some prior π over S and some class of events $\mathcal{T}$. It is easy to see that we can assume, without loss of generality, that $\mathcal{T}$ is such that $\mathcal{T} \ni T \subseteq \hat{T} \implies \hat{T} \in \mathcal{T}$ and $\pi(T) > 0$ for all $T \in \mathcal{T}$. The result will be proved if we can show that there exists a $\theta \in [0,1]$ such that, for any $T \subseteq S$, $T \in \mathcal{T}$ if and only if $\pi(T) \geq \theta$. Fix some event $T \in \mathcal{T}$ and some event $\hat{T} \notin \mathcal{T}$. Now pick $p,q,r \in \Delta(X)$ such that $u(r) < u(q) < u(p)$ and $\pi(T)u(p) + (1-\pi(T))u(r) > u(q)$. Let $f := pTr$ and

[25] To be precise, choose j and λ so that, for $s \in T$, $\lambda(s) := \frac{u(f^\alpha(s))-u(g^\alpha(s))}{1-\lambda^*}$ and $u(j(s)) = \frac{(1-\lambda^*)u(f^\alpha(s))-(u(f^\alpha(s))-u(g^\alpha(s)))}{(1-\lambda^*)-(u(f^\alpha(s))-u(g^\alpha(s)))}$, and, for $s \in S \setminus T$, $\lambda(s) := \frac{u(g^\alpha(s))-u(f^\alpha(s))}{\lambda^*}$ and $u(j(s)) = \frac{\lambda^* u(f^\alpha(s))}{\lambda^*-(u(g^\alpha(s))-u(f^\alpha(s)))}$.

$g := p\hat{T}r$. By the representation in Proposition 1, we must have $\{f\} = c(\{f, g, q\}, q)$. But, by Binary Consistency, this implies that $\{f\} = c(\{f, g, q\}, \diamond)$, which, again by the representation in Proposition 1, implies that $\pi(T) > \pi(\hat{T})$. Notice that T and $\hat{T}$ were entirely generic in the analysis above, so if we define $\theta := \min\{\pi(T) : T \in \mathcal{T}\}$ we have the desired characterization of $\mathcal{T}$. □

A.3. **Proof of Proposition 2.** (2) $\implies$ (1). Suppose $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$ and $f \in A$ are such that $f \in c_1(A, g)$ and $f \in c_2(A, \diamond)$. Since $f \in c_1(A, g)$, we know that $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}_1 \subseteq \mathcal{T}_2$. From $f \in c_2(A, \diamond)$ we know that $f \in \arg\max_{f \in A} \int_S u(f(s))d\pi_2$. But then it is clear from the representation of c_2 that $f \in c_2(A, g)$.

(1) $\implies$ (2). Define the collection of events $\mathcal{T}_2^o$ by $\mathcal{T}_2^o := \mathcal{T}_2 \cup \{T \in \mathcal{T}_1 : \pi_2(T) = 0\}$. Notice that $\int_S u(g(s))d\pi_2 > \int_S u(f(s))d\pi_2$.for any two acts such that $\pi_2(\{s \in S : u(f(s)) \geq u(g(s))\}) = 0$. This implies that $(u, \pi_2, \mathcal{T}_2^o)$ is also a representation of c_2. Now fix any $T \in \mathcal{T}_1$ with $\pi_2(T) > 0$. It is possible to find two acts f and g such that $\int_S u(f(s))d\pi_1 \geq \int_S u(g(s))d\pi_1$, $\int_S u(f(s))d\pi_2 \geq \int_S u(g(s))d\pi_2$ and $\{s \in S : u(f(s)) \geq u(g(s))\} = T$. By the representations of c_1 and c_2, this implies that $f \in c_1(\{f, g\}, g)$ and $f \in c_2(\{f, g\}, \diamond)$. But then (1) implies that $f \in c_2(\{f, g\}, g)$ which can happen only if $T \in \mathcal{T}_2$. But then $\mathcal{T}_1 \cup \mathcal{T}_2 = \mathcal{T}_2^o$, which completes the proof of the proposition. □

A.4. **Proof of Proposition 4.** (a) and (b) are straight forward and the proof is omitted. To see that (c) holds, if u, π and $\theta = 0$ represent c, then c simply maximizes expected utility and $S_c(f) = B_c(f)$ for all $f \in \mathcal{F}$. This shows that (1) implies (3). Also, it is clear that if, for all $s \in S$, $\pi(s) > 0$ implies $\pi(s) \geq \theta$, then u, π and $\theta = 0$ also represent c. That is, (3) implies to (2). So we only have to show that (2) implies (1). Suppose that there exists $s^* \in S$ such that $\theta > \pi(s^*) > 0$. Pick $p, q \in \Delta(X)$ such that $u(p) > u(q)$. Consider the act f such that $f(s^*) = p$ and $f(s) = q$ for all $s \neq s^*$. Notice that $\int_S u(f(s))d\pi > u(q) = \max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}$. Since $S_c(f) \geq \int_S u(f(s))d\pi$ and $B_c(f) \leq \max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}$, we conclude that $S_c(f) > B_c(f)$. □

A.5. **Proof of Proposition 5.** The arguments that show that the representation implies the axioms are routine, so we only show the sufficiency part of the proof. Similar to what we did in the proof of Proposition 1, for any $\lambda : S \to [0, 1]$ and acts f and g, define $f \oplus_{\lambda(.)} g$

to be the act such that $\left(f \oplus_{\lambda(.)} g\right)(s) = \lambda(s)f(s) + (1-\lambda(s))g(s)$ for each $s \in S$. We need the following claim:

Claim 1. *For any acts f, g and h, and $\lambda : S \to (0,1]$, $f \succeq g$ if and only if $f \oplus_{\lambda(.)} h \succeq g \oplus_{\lambda(.)} h$.*

Proof of Claim. Order the states in S in any way. For each $i = 1, ..., |S|$, let $\lambda^i : S \to (0,1]$ be defined by $\lambda^i(s) := \lambda(s)$ if $s \leq i$ and $\lambda^i(s) := 1$ for $s > i$. Now notice that Statewise Independence implies that $f \succeq g$ iff $f \oplus_{\lambda^1(.)} h \succeq g \oplus_{\lambda^1(.)} h$ iff ... iff $f \oplus_{\lambda(.)} h \succeq g \oplus_{\lambda(.)} h$. $\|$

In particular, the claim above implies that the restriction of $\succeq$ to constant acts satisfies the standard Independence axiom. By **B1**, **B2** and **B3**, it is also complete, transitive, continuous and non-trivial. By the expected utility theorem, we know that there exists a non-constant and affine function $u : \Delta(X) \to \mathbb{R}$ such that, for any $p, q \in \Delta(X)$, $p \succeq q$ if and only if $u(p) \geq u(q)$. Without loss of generality we may assume that $u(\Delta(X)) = [0,1]$. Let $\overline{p}$ and $\underline{p}$ be two lotteries such that $u(\overline{p}) = 1$ and $u(\underline{p}) = 0$. Call an event T a candidate for decisiveness if $\overline{p}T\underline{p} \succeq \underline{p}T\overline{p}$. Let $\mathcal{T}$ be the class of all such events. The next two claims show that, for any two acts f and g, $f \succeq g$ if and only if $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.

Claim 2. *If $f, g \in \mathcal{F}$ are such that $u(f(s)) \neq u(g(s))$ for all $s \in S$, then $f \succeq g$ if, and only if, $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.*

Proof of Claim. Suppose the acts f and g are such that $u(f(s)) \neq u(g(s))$ for all $s \in S$. Let $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. Define $\lambda : S \to (0,1]$ by $\lambda(s) := u(f(s)) - u(g(s))$ if $s \in T$ and $\lambda(s) := u(g(s)) - u(f(s))$ if $s \notin T$. Let h be any act such that, for each $s \in T$, $u(h(s)) = \frac{u(g(s))}{1-(u(f(s))-u(g(s)))}$, and, for each $s \notin T$, $u(h(s)) = \frac{u(f(s))}{1-(u(g(s))-u(f(s)))}$. Define $f^\lambda := (\overline{p}T\underline{p}) \oplus_{\lambda(.)} h$ and $g^\lambda := (\underline{p}T\overline{p}) \oplus_{\lambda(.)} h$. By construction, $u(f(s)) = u(f^\lambda(s))$ and $u(g(s)) = u(g^\lambda(s))$ for all $s \in S$. Two applications of Unambiguous transitivity imply that $f \succeq g \iff f^\lambda \succeq g^\lambda$. But, by the previous claim, $f^\lambda \succeq g^\lambda \iff T \in \mathcal{T}$. $\|$

Claim 3. *For any two acts f and g, $f \succeq g$ if, and only if, $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.*

Proof of Claim. Fix any two acts f and g and define $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. By Claim 1, $f \succeq g \iff \frac{1}{2}f + \frac{1}{2}\underline{p} \succeq \frac{1}{2}g + \frac{1}{2}\underline{p}$, so, we can assume, without loss of generality, that $u(f(s)) < 1$ for all $s \in S$. For any $\alpha \in (0,1)$, define $f^{\alpha} := \alpha f + (1-\alpha)\overline{p}$. It is clear that, for α close enough to one, $\{s \in S : u(f^{\alpha}(s)) \geq u(g(s))\} = T$. Moreover, for such α it is clear that $u(f^{\alpha}(s)) \neq u(g(s))$ for all $s \in S$. By the previous claim, we learn that, for all α close enough to one, $f^{\alpha} \succeq g \iff T \in \mathcal{T}$. If $f \succeq g$, then Unambiguous Transitivity implies that $f^{\alpha} \succeq g$ for all $\alpha \in (0,1)$ and, consequently, $T \in \mathcal{T}$. Conversely, if $T \in \mathcal{T}$, then, for all α close enough to one we have $f^{\alpha} \succeq g$. But then Continuity implies that $f \succeq g$. $\|$

The claim above completes the proof of the proposition. $\square$

A.6. **Proof of Theorem 2.** Since $\succeq$ satisfies **B1–B4**, it has a representation as in Proposition 5 for some collection of events $\mathcal{T}$. Suppose $T_1, ..., T_m$, and $\hat{T}_1, ..., \hat{T}_m$ are two finite sequences of events such that $T_i \in \mathcal{T}$ and $\hat{T}_i \notin \mathcal{T}$ for all i. Pick any two lotteries$p, q \in \Delta(X)$ such that $u(p) > u(q)$. The representation in Proposition 5 implies that, for all i, $pT_iq \succeq p \succeq p\hat{T}_iq$. But then **B5** implies that for no $\lambda \in \Delta(m)$ it can be true that $\sum_{i=1}^{m} \lambda_i (p\hat{T}_iq) \trianglerighteq_S \sum_{i=1}^{m} \lambda_i (pT_iq)$. By the representation in Proposition 5, this is equivalent to saying that for no $\lambda \in \Delta(m)$ can it be true that $\sum_{i=1}^{m} \lambda_i \mathbb{1}_{\hat{T}_i} \geq \sum_{i=1}^{m} \lambda_i \mathbb{1}_{T_i}$. That is, the collection $\mathcal{T}$ satisfies the following property:

Property $*$ For any sequences of events $\{T_1, ..., T_m\} \subseteq \mathcal{T}$, $\{\hat{T}_1, ..., \hat{T}_m\} \subseteq 2^S \setminus \mathcal{T}$ and $\lambda \in \Delta(m)$, it cannot be true that $\sum_{i=1}^{m} \lambda_i \mathbb{1}_{\hat{T}_i} \geq \sum_{i=1}^{m} \lambda_i \mathbb{1}_{T_i}$.

Now let $\mathcal{E} := conv\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T} \text{ and } \hat{T} \notin \mathcal{T}\}$.[26] $\mathcal{E}$ is a closed and convex subset of $\mathbb{R}^S$, and, by Property $*$, it is disjoint from $\mathbb{R}^S_-$. Therefore, by the separating hyperplane theorem, there exists a non-null vector $\pi \in \mathbb{R}^S$ such that $\pi \cdot x > 0$ and $\pi \cdot y \leq 0$, for every $x \in \mathcal{E}$ and $y \in \mathbb{R}^S_-$.[27] The vector π must be non–negative, since if there exists $s^* \in S$ for which $\pi(s^*) < 0$, then $\pi \cdot (-\mathbb{1}_{\{s^*\}}) > 0$, in contradiction to the separation by π. Without lost of generality, π can be normalized to be a probability distribution on S. Now note that $\pi \cdot x > 0$ for every $x \in \mathcal{E}$. In particular, $\pi \cdot (\mathbb{1}_T - \mathbb{1}_{\hat{T}}) > 0$ for every $T \in \mathcal{T}$ and $\hat{T} \notin \mathcal{T}$. That is, $\pi(T) > \pi(\hat{T})$ for every $T \in \mathcal{T}$ and $\hat{T} \notin \mathcal{T}$. Thus,

[26] By $conv\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T} \text{ and } \hat{T} \notin \mathcal{T}\}$ we mean the convex hull of the set $\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T}$ and $\hat{T} \notin \mathcal{T}\}$.

[27] For $p, x \in \mathbb{R}^S$ we denote the inner product of p and x by $p \cdot x = \sum_{s \in S} x_s \cdot p_s$.

letting $\theta := \min_{T\in\mathcal{T}} \pi(T)$, we have that $\pi(T) \geq \theta$ if, and only if, $T \in \mathcal{T}$. Finally, as θ is a probability of some event in $\mathcal{T}$, we must have $\theta \in [0,1]$. Since $\emptyset \notin \mathcal{T}$, it must be the case that $\theta > 0$.

Conversely, suppose that $\succeq$ has a representation as in the statement of the theorem. By Proposition 5, we know that $\succeq$ satisfies **B1-B4**. To see that it also satisfies **B5**, pick any three lotteries $p, q, r \in \Delta(X)$ and finite sequences of acts $f_1, ..., f_m$ and $g_1, ..., g_m$ such that, for all $i = 1, ..., m$, $f_i(S) \cup g_i(S) \subseteq \{p, r\}$. Suppose that $\lambda \in \Delta(m)$ is such that $\sum_{i=1}^m \lambda_i g_i \trianglerighteq_S \sum_{i=1}^m \lambda_i f_i$ and $f_i \succeq q$ for all $i = 1, ..., m$. Without loss of generality suppose that $u(p) \geq u(r)$. If $u(r) \geq u(q)$, then the representation implies that $g_i \succeq q$ for all i. If $u(q) > u(p)$, then the representation would imply that it cannot be true that $f_i \succeq q$ for any i. So, the only interesting case remaining is when $u(p) \geq u(q) > u(r)$. For each $i = 1, ..., m$ let $T_i := \{s \in S : u(f_i(s)) = p\}$ and $\hat{T}_i := \{s \in S : u(g_i(s)) = p\}$. By the representation of $\succeq$, it must be the case that $\pi(T_i) \geq \theta$ for $i = 1, ..., m$. Now note that saying that $\sum_{i=1}^m \lambda_i g_i \trianglerighteq_S \sum_{i=1}^m \lambda_i f_i$ is equivalent to say that $\sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i} \geq \sum_{i=1}^m \lambda_i \mathbb{1}_{T_i}$. But this implies that $\sum_{i=1}^m \lambda_i \pi(\hat{T}_i) = \pi \cdot (\sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i}) \geq \pi \cdot (\sum_{i=1}^m \lambda_i \mathbb{1}_{T_i}) = \sum_{i=1}^m \lambda_i \pi(T_i)$. This can be true only if $\pi(\hat{T}_i) \geq \pi(T_i) \geq \theta$, for some $i \in \{1, ..., m\}$, but for such i the representation of $\succeq$ implies that $g_i \succeq q$. $\square$

A.7. **Proof of Proposition 7.** Suppose $\succeq$ is complete and $(\pi, \theta) \in \mathcal{H}_{\succeq}$ is such that $\theta > 1/2$. Suppose that there exists $T \subseteq S$ with $\frac{1}{2} \leq \pi(T) < \theta$ and fix $p, q \in \Delta(X)$ with $u(p) > u(q)$. But observe that the representation of $\succeq$ would imply that pTq and qTp are not comparable. We conclude that $\pi(T) < \frac{1}{2}$ for all $T \subseteq S$ with $\pi(T) < \theta$. But now it is clear that $(\pi, 1/2) \in \mathcal{H}_{\succeq}$. $\square$

A.8. **Proof of Proposition 8.** It is obvious that (2) and (3) are equivalent and, by Proposition 5, it is clear that they imply (1). So, we only need to show that (1) implies (3). By Proposition 5, $\succeq$ has a representation as in Eq. 3. Now take any minimal set $T \in \mathcal{T}$.[28] We now show that $T \subseteq \hat{T}$ for any $\hat{T} \in \mathcal{T}$. To see that, suppose that there exists $\hat{T} \in \mathcal{T}$ such that $T \setminus \hat{T} \neq \emptyset$ and pick two lotteries p and q such that $p \succ q$. Now note that $q \succeq qTp$, $qTp \succeq q(T \cap \hat{T})p$, but, since T is minimal, it cannot be true that $q \succeq q(T \cap \hat{T})p$. We conclude that $T \subseteq \hat{T}$ for any $\hat{T} \in \mathcal{T}$. $\square$

[28]By minimal we mean that there is no $T' \in \mathcal{T}$ such that $T' \subsetneq T$.

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