Framing Effects and Optimization*

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Abstract

We introduce a new model of framing effects associated with where an option appears in a list, whether or not it is the default, and other such easily observable properties. These properties are effortlessly identified by the decision maker, but in contrast, the payoffs to choosing an option can be hard to determine. The decision maker in our model has prior beliefs about the correlation between easily observed properties and payoffs, such as whether items at the top of a list are of higher quality. This decision maker then exerts optimal effort to learn the payoffs to each option, forms appropriate posterior beliefs, and chooses to maximize expected utility. Hence framing effects arise endogenously as an optimal response to payoff uncertainty. We specialize the model to analyze how optimizing behavior may shape the impact of list order on demand. We also show how to recover utility functions for framed choice data, as required for policy analysis. Despite its generality, our model makes falsifiable predictions about the extent of framing effects.

Key Words: Framing Effects, Bounded Rationality, Signal Processing, Revealed Preference, Stochastic Choice, Imperfect Perception, Rational Expectations

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1 Introduction

A multitude of framing effects, in which choice is impacted by how options are described or displayed, have been documented in the psychological and economic literatures. Systematic effects of this form are of ever growing interest to policy makers, because they suggest methods for altering and potentially for improving choices without limiting the available options (see Thaler and Sunstein [2008]). We study those framing effects that derive from easily observed properties of the available options, such as being the default option or at the top of a list.[1] For example, choice probabilities are increased when a given retirement portfolio is the default option (Madrian and Shea [2001]) and when a political candidate is presented at the top of a list (Miller and Krosnick [1998]).[2]

The framing effects that we model arise endogenously as an optimal response to incomplete information about the available choice options. The easily observed properties of a decision environment immediately identify the set of action choices (e.g. select the top item in a list), but the decision maker (DM) starts off uncertain about the payoffs to taking each action. The exact map between actions and prizes in a given decision problem, called the "frame", is unknown to the DM. The DM has prior beliefs about the likelihood of all possible frames, and by exerting perceptual effort, learns better the actual frame faced in a given decision problem. From this learning, the DM forms posterior beliefs, and then chooses the action that maximizes expected utility.

While boundedly rational, the agents in our model are sophisticated about how these bounds constrain them, doing as well as possible in light of their unavoidable limitations. Specifically, each DM selects an optimal strategy balancing the costs of perceptual effort against the resulting improvement in expected prize utility. Moreover, final choices are selected optimally given the DM's posterior understanding of available options. Because there may be remaining uncertainty even after exerting optimal effort, final choice is generally among lotteries over prizes rather than among deterministic prizes.

An insight from the model is that framing effects are strongly shaped by the DM's beliefs about the correlation between the easily observed properties of an option and the utility of that option.[3]

[1] We do not examine, for instance, those framing effects that arise from how outcomes are depicted (e.g. the Asian disease problem of Tversky and Kahneman [1981]).

[2] See Salant and Rubinstein [2008] for more examples of this type.

[3] That beliefs are natural to consider in strategic models of attentionally limited choice is shown by Eliaz and

By way of example, when options are listed by Google or Amazon, many searchers start with the first item in the list (see Athey and Ellison [2009]). Yet if the first item or two in these lists were to be dominated by low quality algorithmic "cheats", we would expect searchers to notice this and then to start searching further down the list. Similarly, if a company gained a reputation for using defaults to dupe its customers into choosing inferior products, we might expect an increasing number of opt outs (see Brown and Krishna [2004]). On the other hand, if individuals believe that policy makers use high quality defaults, they may opt out only infrequently, particularly if items are complex (see Caplin and Martin [2012]).

We illustrate the main elements of our model in the special case of ordered search through a list of options, as in Rubinstein and Salant [2006]. In addition to modeling choice, we address the recoverability question posed by Rubinstein and Salant [2011]: if we observe boundedly rational choices that result from optimizing behavior, to what extent can we recover the underlying utility function? This question, which is vital from the viewpoint of policy design, is made non-trivial by the fact that framing effects destroy the automatic link between preferences and choices: one item may be chosen over another in a particular frame even if it is of strictly lower utility. We show that the model enables us in principle to recover information not only concerning pure prize preferences, but also concerning the relative strength of preference for one prize over another.

In addition to our in-depth analysis of list order search, we provide a far-reaching generalization based on a "perceptual mapping" that maps objective frames into subjective states. These states reflect the DM's uncertainty concerning the frame they face (their "perception" of the frame). We assume expectations are rational, as in models of rational inattention (e.g. Sims [2003], Gabaix [2012], Woodford [2012]). This means that DMs have correct beliefs about how likely the available choices are to yield the possible prizes given their subjective state. That is, given all the frames that could have put the DM in a particular subjective state, the lottery associated with each action is correct on average. This approach is analogous to a Bayesian signal process in the mind of the DM. We illustrate how this regularity links our model to those of Eyster and Rabin [2005], Jehiel [2005], and Bolton and Faure-Grimaud [2009], yet differentiates it from such models as Esponda [2008], Steiner and Stewart [2008], and Schwartzstein [2012].

Spiegler [2011a, b]. More broadly, McKenzie and Nelson [2003] give evidence that verbal descriptions impact beliefs through their informational content, while Kamenica [2008] presents a model in which context impacts beliefs through its informational content.

Given the generality of this model of framing effects, it is not immediately obvious that it is testable when perception is unobservable. We derive positive results in this regard. Just as the model of objective utility maximization is equivalent to a set of linear constraints (Afriat [1967]), so our model of subjective optimization is equivalent to a set of "No Improving Action Switches" constraints. These constraints indicate whether a data set that exhibits framing effects is consistent with our general model. Further, these constraints can be used to discover the extent to which the underlying utility function can be recovered when framing effects are present, but perception is unobservable.

In terms of methodology, our approach follows up on the proposals of Caplin and Dean [2011] to incorporate non-standard data in models of choice (see Gomberg [2011] for an analysis of committee voting along these lines) and of Lipman [1991] to treat boundedly rational agents as constrained optimizers.

In section 2, we outline the action/prize distinction that sits at the heart of our model. In sections 3 and 4, we analyze list order search. In sections 5 and 6, we present the general model, and we highlight similarities and differences with alternative approaches in section 7. There are close connections both with models of rational inattention and with various strategic models such as that of Bergemann and Morris [2011]. The applicability of our framework to policy questions is noted in the concluding remarks in section 8.

2 Model

2.1 Prizes and Actions

There is a finite prize set X of size $N \geq 2$, from which the DM is choosing, with generic element $x_n \in X$. We allow this set to be presented in many different manners. To formalize presentational differences, we distinguish the act of choice from the receipt of a prize. The choice environment has easily observable properties, such as precise physical locations on a screen, placements in a store, etc., which are internalized without difficulty. From these properties, the DM knows immediately the set of available actions, such as choosing the top item on the screen, the leftmost item on a shelf, etc.

Definition 1 *An* ***action set*** *for prize set X is a finite set A that has at least the same cardinality as the prize set,*

$$|A| = M \geq |X| = N \geq 2.$$

The generic element of action set A is $a_m \in A$. For much of the remaining paper, we operate not only with a fixed set of prizes, but also with a fixed action set.

2.2 Frames

Choosing an action will yield one of the prizes, as formalized in a particular mapping of actions to prizes. Technically, a **frame** for prize set X is an onto function $f : A \to X$, with $\mathcal{F}$ the set of all such frames,

$$\mathcal{F} = \{f : A \to X | f \text{ is onto}\}.$$

To illustrate, consider a case in which three prizes are presented in one of three positions in a list. Table 1 presents all the possible manners in which the prizes may be framed. By way of interpretation, note that action a_1, corresponding to picking the first position on the list, yields prize x_1 when the frame is either f_1 or f_2.

Table 1. Example frames (3 actions, 3 prizes)

	frames $\mathcal{F}$					
actions A	f_1	f_2	f_3	f_4	f_5	f_6
a_1	x_1	x_1	x_2	x_2	x_3	x_3
a_2	x_2	x_3	x_1	x_3	x_1	x_2
a_3	x_3	x_2	x_3	x_1	x_2	x_1

The assumption that frames are onto captures the idea that the entire prize set is in fact available in every choice problem.[4] Thus, from the viewpoint of classical choice theory, frames should be irrelevant, since the feasible set of prizes is invariant. In many of the cases that we consider, including the case of top-down search, we further restrict the function $f : A \to X$ to be one-to-one, as in table 1. However, allowing for the more general case enables the model to cover

[4] This assumption is not required in the formal treatment of the model. We introduce it to clarify the application to framing, which involves distinct presentations of one and the same set of prizes.

such phenomena as "needle in a haystack" choice sets, in which one good prize is obscured by the availability of several equivalent bad prizes.

2.3 Expectations

We model a DM making many choices from the same fixed sets of prizes and actions. While the prize and action sets remain fixed, the frame, which determines the prize that each action yields, is randomly determined in each decision problem based on a specific **frame-generating mechanism** (a probability measure over frames) $\mu \in \Delta(\mathcal{F})$. For example, if the DM was equally likely to face all frames in any given decision problem, then the frame-generating mechanism for table 1 would be,

$$\mu(f_1) = \mu(f_2) = \mu(f_3) = \mu(f_4) = \mu(f_5) = \mu(f_5) = \frac{1}{6}.$$

In our model, the DM knows the available actions A, the possible prizes X, the set of frames $\mathcal{F}$, and the probability that μ assigns to each frame, but does not know the frame f realized in each decision problem. Because of this uncertainty concerning how actions get translated into prizes, there is room for misperception and mistakes.

The measure μ provides the DM a statistical sense of what happens if a given action is selected, so it can be thought of as the DM's (stable) prior beliefs about the payoffs to each action before any decision problem. After learning something about which frame was realized in a given decision problem (summarized by information $\mathcal{I}$), the DM arrives at posterior beliefs $\mu^{\mathcal{I}}$. Continuing the example above, assume that before making a choice, the DM learns only that the prize associated with action a_1 was x_1. The DM's posterior beliefs would be,

$$\begin{aligned} \mu^{\mathcal{I}}(f_1) &= \mu^{\mathcal{I}}(f_2) = \frac{1}{2}, \\ \mu^{\mathcal{I}}(f_3) &= \mu^{\mathcal{I}}(f_4) = \mu^{\mathcal{I}}(f_5) = \mu^{\mathcal{I}}(f_6) = 0. \end{aligned}$$

2.4 Utility and Choice

We model the DM as an expected utility maximizer with utility function $U : X \longrightarrow \mathbb{R}$. In addition we assume that expectations are rational. In this case, a useful way to compute expected utility is with the posterior probability $s^{\mathcal{I}}_{mn}$ that action a_m will yield prize x_n given information $\mathcal{I}$,

$$s^{\mathcal{I}}_{mn} = \mu^{\mathcal{I}}\{f \in \mathcal{F} | f(a_m) = x_n\}.$$

Optimal choice of action reflects this posterior,

$$C\left(A|\mathcal{I}\right) \in \arg\max_{a_m \in A} \sum_{n=1}^{N} s_{mn}^{\mathcal{I}} U(x_n).$$

3 List Order Search and Optimization

We apply our framework to model DMs engaged in list order search to depth J, where $1 \leq J < M$. In this context, searching the first J positions means fully uncovering the prizes associated with actions a_1 to a_J. To pin down the model's implications for choice, we use list order to break ties in cases of indifference. That is, if the expected utility of two action choices is equivalent, the action closer to the top of the list is chosen.

For simplicity, we consider the case with three prizes and three actions. In this simple case, we can represent each frame by a permutation $q = (q_1, q_2, q_3)$ of the prizes. The interpretation of permutation q is that prize $q_m \in X$ is at position m in the list, and hence associated with action a_m. For example, if $q_3 = x_2$ then prize x_2 is in the 3rd position in the list, and thus associated with action a_3.

We analyze below three distinct respects in which observed framing effects are impacted by optimization. We first characterize optimal choice if only the top position in the list is searched. We then consider the optimality of searching some other item than the top position in the list. Finally, we consider the fully optimal search procedure, which is sequential, so that the stopping rule depends on the prizes observed in previously searched positions.

3.1 Optimal Choice after Searching Top Position

Consider a simple case in which $J = 1$, so that only the top position in the list is searched. As a result, the prize corresponding to action a_1 is always seen with perfect clarity, but the prizes corresponding to actions a_2 and a_3 are not seen at all. In addition, we set $X = \{\$1, \$2, \$3\}$ and assume that the DM is strictly risk averse with monotonic utility for money.

We first examine optimal choice if each of the 6 possible frames is equiprobable. Note that if a_1 is seen to yield a prize of just \$1 (when $q_1 = \$1$), then either of the other two actions has maximal expected utility of $\frac{1}{2}U\left(\$3\right) + \frac{1}{2}U\left(\$2\right)$, so that the DM will choose action a_2 given our order based tie-breaking rule. On the other hand, if $q_1 \in \{\$2, \$3\}$, then a_1 will be chosen. What results is a clear framing effect: the chosen action depends on which prize is in the searched location.

Optimal choice is impacted not only by the frame, but also by the frame generating mechanism. To see this, consider the same prize and action sets, but assume that there are asymmetric frame probabilities:

$$\begin{aligned}
\mu(\$1, \$2, \$3) &= \mu(\$2, \$3, \$1) = \frac{3}{12}, \\
\mu(\$3, \$1, \$2) &= \mu(\$3, \$2, \$1) = \frac{2}{12}, \\
\mu(\$2, \$1, \$3) &= \mu(\$1, \$3, \$2) = \frac{1}{12}.
\end{aligned}$$

A priori, choosing a_2 is more appealing than choosing a_3 because it has a higher expected utility. If \$2 is seen in the first position, then this ranking is preserved. However, if \$1 is seen in the first position, then the ranking is reversed: a_3 is more appealing than a_2.

This reversal in the rankings of unexplored alternatives has implications for final choices. For example, if a_1 is seen to yield a prize of just \$1 (when $q_1 = \$1$), then the expected utility of each action is,

$$\begin{aligned}
&U(\$1) \text{ for } a_1, \\
&\frac{1}{4}U(\$3) + \frac{3}{4}U(\$2) \text{ for } a_2, \\
&\frac{3}{4}U(\$3) + \frac{1}{4}U(\$2) \text{ for } a3.
\end{aligned}$$

This implies that the DM will choose action a_3. If instead a_2 is seen to yield a prize of \$2 (when $q_1 = \$2$), then the expected utility of each action is,

$$\begin{aligned}
&U(\$2) \text{ for } a_1, \\
&\frac{3}{4}U(\$3) + \frac{1}{4}U(\$1) \text{ for } a_2, \\
&\frac{1}{4}U(\$3) + \frac{3}{4}U(\$1) \text{ for } a_3.
\end{aligned}$$

In this case a_2 will be chosen by agents with low levels of risk aversion, while a_1 will be chosen by those with high enough levels of risk aversion.

3.2 Optimal Search Order

A DM facing the above environment can work out the expected utility from searching any single position, and as such determine which position is optimal to search if just one position can be searched. For the asymmetric example above, note that actions a_1, a_2, and a_3 are all chosen with probability $\frac{1}{3}$ for low levels of risk aversion. With regard to the resulting prize lotteries, when a_1 is chosen it yields \$3 for sure, when a_2 is chosen it yields $\frac{3}{4}U(\$3) + \frac{1}{4}U(\$1)$, and when a_3 is chosen it yields $\frac{3}{4}U(\$3) + \frac{1}{4}U(\$2)$. Overall, the expected utility of searching the first position is,

$$\frac{10}{12}U(\$3) + \frac{1}{12}U(\$2) + \frac{1}{12}U(\$1).$$

Straightforward computations show that when risk aversion is low, searching the second position instead of the first position results in choice of a_2 when $q_2 = \$3$, choice of a_3 when $q_2 = \$2$, and choice of a_1 when $q_2 = \$1$. As a result, when a_2 is chosen it yields \$3 for sure, when a_3 is chosen it yields $\frac{3}{5}U(\$3) + \frac{2}{5}U(\$1)$, and when a_1 is chosen it yields $\frac{2}{3}U(\$3) + \frac{1}{3}U(\$2)$. Weighting up these rewards by their probabilities, we identify expected utility from searching the middle position as,

$$\frac{9}{12}U(\$3) + \frac{1}{12}U(\$2) + \frac{2}{12}U(\$1).$$

We conclude in this case that searching the top position is strictly superior to searching the middle position. Continuing in the analogous manner one can confirm also that searching the top position is superior to searching the bottom position as well.

Note that optimal search order depends crucially on expectations. In this example, searching the first position is valuable because it reveals significant information concerning the location of the \$3 prize. If we swap the values of q_1 and q_3 in the frame-generating mechanism, then it is optimal instead to search the bottom position. Note also that the process above is completely general. Given any search order and any given depth of search J, optimal choice and the prize received are a deterministic function of the frame. The end result is that the expected utility of any search order can be computed in mechanical fashion, revealing optimal search order.

3.3 Optimal Sequential Search

Use of our model to solve for the optimal search strategy extends beyond fixed search to depth J. One can, for example, solve for the optimal sequential search strategy when the cost of search in expected utility units is $C > 0$ (as in Gabaix and Laibson [2005] and Caplin, Dean, and Martin [2011]).[5] This is simplest to illustrate when all frames are equiprobable and DMs search in list order. We further simplify the example by assuming that the DM is risk neutral.

The optimal sequential search strategy can be computed by standard backward inductive logic. If the top two positions have been searched, the action associated with the best prize is identified for sure, yielding deterministic prize \$3. We now compute the continuation value associated with the optimal strategy if just the top position has been searched, which we denote $V_1(q_1)$ and measure in terms of expected dollars. Note first that it is clearly optimal to stop and save any additional search cost if $q_1 = \$3$, so that,

$$V_1(\$3) = 3.$$

If $q_1 = \$2$, then action a_1 would be chosen if search stopped, yielding a deterministic \$2 prize. Hence additional search (searching the second position) will be worthwhile if and only if $C \leq 1$,

$$V_1(\$2) = \begin{cases} 2 \text{ if } C > 1; \\ 3 - C \text{ if } C \leq 1. \end{cases}$$

Finally, if $q_1 = \$1$, then the best option with no additional search is to take action a_2 and thereby to receive a 50% chance of \$2 and a 50% chance of \$3. This is worthwhile if and only if $C \leq 0.5$. Hence,

$$V_1(\$1) = \begin{cases} 2.5 \text{ if } C > 0.5; \\ 3 - C \text{ if } C \leq 0.5. \end{cases}$$

With this, we can compute the conditions under which the initial search (searching the first position) is worthwhile. If the initial search does take place, it is equally likely to yield each possible value of $V_1(q_1)$, so that the expected value of searching the first position, defined as EV_1, can be straightforwardly computed:

$$EV_1 = \begin{cases} 2.5 - C \text{ if } C > 1; \\ \frac{17}{6} - \frac{C}{3} - C \text{ if } C \in (0.5, 1]; \\ 3 - \frac{2C}{3} - C \text{ if } C \in (0, 0.5]. \end{cases}$$

If no search occurs, then the optimal choice is to pick a_1 and get each prize with probability $\frac{1}{3}$ for an expected reward of \$2. Hence we conclude that it is strictly optimal not to search at all if $C > 1$

[5] The ability to use this model to compute optimal sequential search strategies generalizes to an arbitrary cost function specified in expected utility terms.

and strictly optimal to undertake the first search if $C < 0.5$. There is a unique cutoff level of cost $\bar{C} \in (0.5, 1]$ at which there is indifference,

$$\frac{17}{6} - \frac{\bar{C}}{3} - \bar{C} = 2;$$

so that $\bar{C} = \frac{5}{8}$. Thus, searching the top position is optimal if and only if the cost of search is $\frac{5}{8}$ or below.

4 Recoverability

In standard choice theory, choice of prize x_1 when x_2 is available indicates for sure that x_1 has utility at least as high as x_2. This simple condition fails in the case of list order search, as illustrated in the examples above in which the \$2 prize is chosen despite the known availability of the superior \$3 prize. Hence, as Rubinstein and Salant [2011] point out, the extent to which utilities can be recovered from choices when there is list order search depends on what is learned during the course of the search process. We now address this recoverability question in the context of the above examples, highlighting in particular the role that expectations play.

There is one key feature of our model that provides us with traction in recovering the utility function. Given that beliefs are consistent, the prize lottery that is chosen must be at least as good as the prize lotteries associated with alternative choices. This limits the set of utility functions that can give rise to a given pattern of choices, providing the answer in our model to the question of recoverability. What makes the resulting inequalities non-standard is that they provide information not only on the ranking of pure prizes, but also on preferences over lotteries.

To illustrate how to recover the set of consistent utility functions, we use the asymmetric frame generating mechanism of the last section, but generalize the prizes to $X = \{x_1, x_2, x_3\}$ so that ex ante there is no known structure to preferences over prizes:

$$\begin{aligned}
\mu(x_1, x_2, x_3) &= \mu(x_2, x_3, x_1) = \frac{3}{12}; \\
\mu(x_3, x_1, x_2) &= \mu(x_3, x_2, x_1) = \frac{2}{12}; \\
\mu(x_2, x_1, x_3) &= \mu(x_1, x_3, x_2) = \frac{1}{12}.
\end{aligned}$$

In addition, we assume that the action choices observed in each frame are: a_1 when $q_1 = x_3$; a_2 when $q_1 = x_2$; and a_3 when $q_1 = x_1$.

Given this frame generating mechanism and pattern of choices, what we can infer about utility for x_1, x_2, and x_3? Given that switching to alternative choices can never raise utility, utility levels must satisfy a particular set of linear inequalities. We display these inequalities for each of the three action choices, contrasting the lottery that is derived from choosing a particular action to those available from the alternative action choices.

Starting with a_1, the first such inequality asserts that the expected utility of choosing a_1 should weakly dominate the expected utility from choosing a_2 when a_1 was selected ($q_1 = x_3$),

$$U(x_3) \geq \frac{1}{2}U(x_2) + \frac{1}{2}U(x_1).$$

An additional, but identical, inequality states that the expected utility of choosing a_1 should weakly dominate the expected utility from choosing a_3 in the frames where a_1 was selected. These inequalities are not strict given that the unchosen actions are further down the list and that list order is used to break ties.

The inequality associated with choice of a_2 over a_1 states that a_2 should be strictly dominant in the frames it was selected, while the corresponding inequality for action a_2 over a_3 involves only weak dominance,

$$\begin{aligned} \frac{3}{4}U(x_3) + \frac{1}{4}U(x_1) \quad &> \quad U(x_2); \quad (*) \\ \frac{3}{4}U(x_3) + \frac{1}{4}U(x_1) \quad &\geq \quad \frac{1}{4}U(x_3) + \frac{3}{4}U(x_1). \end{aligned}$$

Finally, with regard to a_3, the inequalities state that the lottery choice associated with a_3 in the frames in which it is selected should strictly dominate the lotteries associated with a_1 and a_2,

$$\begin{aligned} \frac{3}{4}U(x_3) + \frac{1}{4}U(x_2) \quad &> \quad U(x_1); \quad (**) \\ \frac{3}{4}U(x_3) + \frac{1}{4}U(x_2) \quad &> \quad \frac{1}{4}U(x_3) + \frac{3}{4}U(x_2). \quad (***) \end{aligned}$$

From $(***)$ we can infer that $U(x_3) > U(x_2)$, which combines with $(**)$ to imply that $U(x_3) > U(x_1)$. Yet this does not allow us to make a direct inference on the preference as between pure prizes x_1 and x_2. However, from $(*)$ and $(**)$, we can find two additional inequalities that place a lower bound on the utility advantage of x_3 over x_2 and x_1,

$$U(x_3) > \max\left\{\frac{4U(x_1) - U(x_2)}{3}; \frac{4U(x_2) - U(x_1)}{3}\right\}.$$

The above inequalities are both necessary and sufficient for expected utilities to rationalize the observed behavior in our model. They illustrate how the model enables us to restrict the relative strength of preferences for one prize over another. This feature is general, as established in section 6.

4.1 Depth $J \geq 2$

If $J \geq 2$, recoverability can be achieved by combining standard revealed preference techniques with probabilistic constraints such as those above. When one of the top J actions is picked, we can infer that the actual prize it yields is known to the DM, so it must strictly dominate all prizes above it in the list and weakly dominate all prizes *seen* below it on the list (up to the Jth position). This implies that much can be inferred about pure prize preferences using standard revealed preference techniques (as developed by Samuelson [1938] and Richter [1966]). Moreover, we can infer that the chosen prize weakly dominates the uncertain prize lottery associated with each of the unexplored positions.

To give a simple example, if all permutations are possible and the chosen action is always one of the top J actions, then by standard revealed preference techniques the complete preference ordering can be inferred over all prizes ever chosen, with the additional comment that all unchosen prizes must be of strictly lower utility than any prize ever chosen. At the same time, the probabilistic constraints shed light on the utility advantage of the chosen prizes over the unchosen prizes.

5 General Model

The finite choice set X, finite action set A, frame-generating mechanism $\mu \in \Delta(\mathcal{F})$, and expected utility function $U : X \longrightarrow \mathbb{R}$ are key components of the general model. There are four additional elements that we specify in this section: a **subjective state space** S; a set of **perceptual mappings** Π that map frames into simple lotteries over the subjective state space; a **perceptual cost function** K that associates utility costs with these perceptual mappings; and a **choice function** C that maps possible subjective states into action choices.

While it may seem non-standard for us to focus on subjective states in a model of framing effects, our approach aligns closely to standard models of Bayesian signal processing. With the assumption of rational expectations, we show below that one can effectively interpret the model as involving Bayesian signal processing on the part of the DM based on a set of subjectively observed signals.

5.1 Subjective States

The set S comprises all subjective mental states of the DM. In general, S captures all information that DMs extract from the choice environment and the specific frame in front of them. More specifically, S may encode characteristics or facets of the available goods, the state of a finite automaton, etc. As in the Savage framework, this state is used to encapsulate the DM's uncertainty about the consequences of available choices.

As previewed in section 2, we set $S = \Delta(X)^M$, the space of ordered lists of M subjective prize lotteries. Let s_{mn} be the probability that action choice a_m results in prize x_n, so that,

$$S = \{s \in \mathbb{R}_+^{MN} | \sum_{n=1}^{N} s_{mn} = 1 \text{ for all } m \in \{1, \dots, M\}\}.$$

The reason to so limit the subjective state space is that we model choice using expected utility theory, for which lotteries over prizes are the appropriate objects of choice.

5.2 Perceptual Mappings and Rational Expectations

A perceptual mapping π links possible frames with lotteries over subjective states.

Definition 2 *A perceptual mapping is a function from $\mathcal{F}(\mu)$, the support of μ, to $\Delta(S)$, the probability distributions over S with finite support,*

$$\pi : \mathcal{F}(\mu) \rightarrow \Delta(S).$$

We let $S(\pi) \subset S$ denote the support of π, noting that the finiteness of $\mathcal{F}(\mu)$ is inherited by $S(\pi)$.

Note that in the list order example above, the perception function is deterministic because search order is fixed and each position is searched fully. By allowing for stochasticity, we allow for cases in which search takes place in a random order and for partial search of a position.

Not all perceptual mappings are feasible in our model. We assume that expectations are rational, so that perceptual states reflect reality in a statistical sense. That is, given all the frames that could have put the DM in a particular subjective state, the lottery associated with each action is correct on average. In what follows, we treat rational expectations as a constraint on the perceptual mapping.

Definition 3 *Given $\mu \in \Delta(\mathcal{F})$, perceptual mapping $\pi : \mathcal{F}(\mu) \rightarrow \Delta(S)$ satisfies **rational expectations** if, for all $m \in \{1, \ldots, M\}$, $n \in \{1, \ldots, N\}$, and $s \in S(\pi)$,*

$$s_{mn} = \frac{\sum_{\{f \in \mathcal{F}(\mu) | f(a_m) = x_n\}} \mu(f) \pi^f(s)}{\sum_{\{f \in \mathcal{F}(\mu)\}} \mu(f) \pi^f(s)}.$$

We let $\Pi^{RE}(\mu)$ denote the set of all perceptual mappings that satisfy rational expectations.

Figure 1 illustrates the restrictions implied by rational expectations in a $2 \times 2 \times 2$ case (2 prizes, 2 actions, and 2 frames). The actions are $a_{1,2}$, the prizes are $x_{1,2}$, and the frames are $f_{1,2}$. In frame f_1, a_1 produces x_1, and a_2 produces x_2. On the other hand, in frame f_2, a_1 produces x_2, and a_2 produces x_1.

The decision tree provides a representation of all objective states of the world as well as the subjective states of mind. The first set of nodes record the stochastic structure of the frame generating mechanism μ, while the subsequent set correspond to the stochastic structure of the subjective states, s^1 and s^2. The subjective states are connected with dashed lines, which represent the information sets of the DM. Each node has a black edge that goes to the action that is adopted, action a_i in state s^i, and also a grey edge corresponding to the untaken alternative action.

The figure shows how rational expectations identify the states s^1 and s^2. Letting s^k_{ij} for $k = 1, 2$ denote the probability that action a_i will produce prize x_j in state s^k, this condition can be stated as,

$$s^k_{11} = s^k_{22} = 1 - s^k_{21} = 1 - s^k_{12} = \frac{\mu(f_1) \cdot \pi^{f_1}(s^k)}{\mu(f_1) \cdot \pi^{f_1}(s^k) + \mu(f_2) \cdot \pi^{f_2}(s^k)},$$

where $\pi^{f_i}(s^k)$ denotes the probability in frame f_i that the subjective state will be s^k. The figure illustrates that there is a natural interpretation of rational expectations as reflecting some signal process that is subjectively available to the DM. In that manner, one can view the DM as a classical Bayesian.

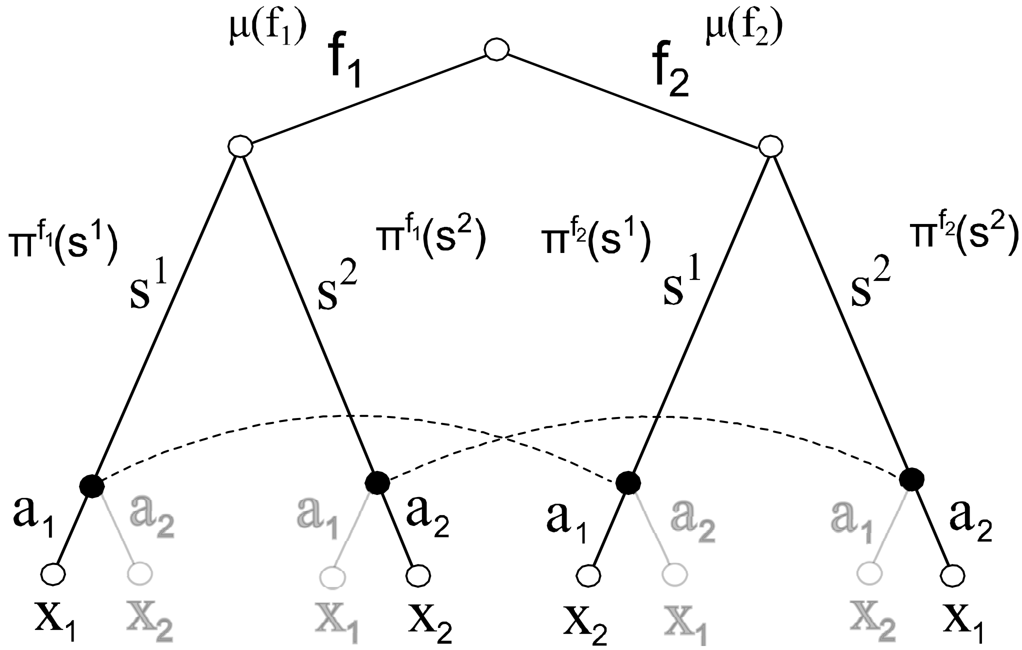

Figure 1: Decision Tree for DM

A non-standard feature of our model of framing effects is that each subjective signal may contain information on the prizes associated with both available actions. It is more standard to model information as action specific, as in Bolton and Faure-Grimaud [2009], so that signals reveal information on just one available prize at a time. Caplin and Martin [2012] develop a Bayesian model of attentional effort that incorporates the multi-faceted nature of subjective signals that our model captures.

5.3 Perceptual Costs

In our optimizing framework, the perceptual mapping is chosen by the DM based on underlying costs. For example, in the case of sequential list order search, the depth of search was determined based on a search cost function that was separable from the utility function over prizes. Likewise, Caplin and Martin [2012] solve for optimal attentional choice when there are separable costs of attentional effort and attention can improve the accuracy of perception. This same approach can be adopted with great generality to solve for optimal perceptual strategies in a wide class of models. To carry out this optimization, one must specify a cost function on a set of feasible perceptual strategies. To simplify, we model perceptual costs in the general case as separable from prize utility and as having to be chosen ex ante, as in the case of batch search.[6]

In formal terms, the key additional model element is a perceptual cost function measured in expected utility units,

$$K : \Pi^{RE}(\mu) \longrightarrow \mathbb{R}_+.$$

As noted, we assume below that prize utilities and perceptual costs are separable, and that net expected utility is the difference between the expected prize utility and this psychic cost.

5.4 The Subjective Choice Function and Stochasticity

While the set $S(\pi)$ serves as the natural domain of the subjective choice function given $\pi \in \Pi^{RE}(\mu)$, it is important for the definition of perceptual optimality to allow for an unrestricted domain,

$$C : S \longrightarrow A.$$

Note that the above formulation implies that choices are based solely on the state of uncertainty at the moment of choice, not on the process of perception that generated that state.

Our assumption that the choice function is deterministic mirrors the literature on finite automata (see Osborne and Rubinstein [1994], p.140). Thus, all randomness in choice behavior for a given frame is due to stochasticity in perception (as in Sims [2003] and Woodford [2012]). Note that this form of stochasticity is quite different than that produced in standard models in which all randomness is placed in the utility function (see Luce [1959] and Block and Marschak [1960]). The most famous axiom for such cases is Luce's axiom, whereby the ratio of the probabilities of choosing any one item over any other is independent of other "irrelevant" alternatives. While this has been relaxed in many ways, most stochastic utility models place restrictions on the relative probabilities of choosing one option over another across distinct choice sets. Restrictions of this form will not apply in our model, since the utility function is fixed and the source of stochasticity is incomplete comprehension of available options.

[6] Nothing changes in principle if one allows for sequential search. The complication is largely notational: one has to build up the recursive machinery to define the costs of perceptual strategies that are sequential in nature.

5.5 Optimal Final Choice and Optimal Perception

Pulling together the above strands, note that the inputs to our framing model are sets X and A, measure μ, expected utility function U, and perceptual cost function K. A model of optimal perception and choice then comprises a perceptual strategy $\hat{\pi} \in \Pi^{RE}(\mu)$ and a corresponding choice function $\hat{C} : S \longrightarrow A$ such that final choices are utility maximizing and the perceptual strategy maximizes the difference between expected prize utility and perceptual cost.

Definition 4 *A perceptual strategy $\hat{\pi} \in \Pi^{RE}(\mu)$ and choice function $\hat{C} : S \longrightarrow A$ are optimal for framing model (X, A, μ, K, U) if they satisfy:*

1. *Perceptual Optimality,*

$$\hat{\pi} \in \arg \max_{\pi \in \Pi^{RE}(\mu)} \left[\sum_{f \in \mathcal{F}(\mu)} \sum_{s \in S} \mu(f) \pi^f(s) U(\hat{C}(s)) - K(\pi) \right].$$

2. *Final Decision Optimality,*

$$\hat{C}(s) \in \arg \max_{1 \le m \le M} \sum_{n=1}^{N} s_{mn} U(x_n),$$

for all $s \in S$.

After substitution for s based on rational expectations, this choice function bears a resemblance to that in the Anscombe-Aumann framework (see Kreps [1988], p. 38 for an overview), but in our case the probability of each prize in each state is generated through the frame.

Note that $U(\hat{C}(s))$ is independent of which optimal action is selected. Hence the conditions reduce to selecting the perceptual mapping to maximize net expected utility given rational expectations. Given that $\Pi^{RE}(\mu)$ is not in general finite, continuity conditions are required to guarantee existence.

6 Recoverability and Falsifiability with Unobservable Perception

In the case of list order search, the mode of search and the associated costs were treated as known to the model builder, so optimal perception was mechanically computed. Thus, how choice varies with optimal perception was directly observed. As a result, testing for consistency and recovering consistent utility functions was straightforward. With the general model, we can adopt a broader approach in which the form of perception and the associated costs are unknown to the model builder, so that optimal perception cannot be determined. Thus, how choice varies with optimal perception is no longer directly observable, and we are left with more unknowns to infer.

While perception is unobservable, there are general model elements that might be visible to an idealized outside observer, such as an econometrician or experimental designer: the prize set; the action set; the frame-generating mechanism; and the (possibly stochastic) actions chosen in each frame. Given these elements, we pose two related questions. First, is our model of optimal framing effects vacuous, or does it in some manner restrict the nature of these effects? Second, we address the recoverability question of section 3 when perception is unobservable. One might expect recoverability conditions to be more intricate in this case, given the relative paucity of data. The main result of this section is that, to the contrary, the recoverability conditions are precisely analogous to the conditions derived already in the case of list order search. Moreover, these same conditions can be used to fully characterize the sense in which the general model is falsifiable.

6.1 The Ideal Data Set and Framing Effects

As indicated above, an ideal outside observer cannot condition action choices on a known perceptual strategy, but can condition them on the observed frame. Given X, A, and $\mu \in \Delta(\mathcal{F})$, an **ideal data set** (IDS) P identifies the probability distribution over action choices as it depends on the frame,

$$P : \mathcal{F}(\mu) \longrightarrow \Delta(A),$$

where $\mathcal{F}(\mu) \subset \mathcal{F}$ is the support of μ.[7]

This data set allows us to cleanly identify the existence of framing effects. Technically, an IDS exhibits a framing effect if for any $x_n \in X$, there exists frames $f, g \in \mathcal{F}(\mu)$ such that,

$$P^f\left(\{a_m \in A | f(a_m) = x_n\}\right) \neq P^g\left(\{a_m \in A | g(a_m) = x_n\}\right),$$

with the convention that $P^f(B)$ is the probability of action set $B \subset A$ being chosen in frame f.

[7] The domain of this data does not allow the order of observations to be recorded, hence we will not consider explanatory hypotheses that involve learning about the stochastic structure of the environment.

6.2 Optimal Framing Representation

Our recoverability question concerns whether and how one can rationalize in our model any particular set of observables (X, A, μ, P).

Definition 5 *(X, A, μ, P) has an **optimal framing representation** if there exists $\bar{K} : \Pi^{RE}(\mu) \to \mathbb{R}_+, \bar{U} : X \to \mathbb{R}$, $\bar{\pi} \in \Pi^{RE}(\mu)$, and $\bar{C} : S \longrightarrow A$, satisfying:*

1. *Data Matching: $P^f(a_m) = \bar{\pi}^f(\bar{C}^{-1}(a_m))$ for all $f \in F(\mu)$ and $a_m \in A$.*
2. *Optimality: $\bar{\pi} \in \Pi^{RE}(\mu)$ and $\bar{C} : S \longrightarrow A$ are optimal for framing model $(X, A, \mu, \bar{K}, \bar{U})$.*
3. *Non-Triviality: there exists $j, k \in \{1, \ldots, M\}$ and $s \in S(\bar{\pi})$ such that,*
$$\sum_{n=1}^{N} s_{jn} U(x_n) > \sum_{n=1}^{N} s_{kn} U(x_n).$$

Condition 1 requires that together the perceptual mapping and the choice function explain the observed data. Condition 2 requires that the perceptual mapping and choice function are optimally chosen. Non-triviality prevents the conditions from being satisfied by a utility function in which all actions always yield identical utility.

6.3 The NIAS Lemma

The central characterization result is that a necessary and sufficient condition for an IDS to have an optimal framing representation is that there exists a utility function such that, for any given action, it is better not to switch to taking some fixed alternative action in all situations in which that action was taken. To ensure non-triviality, the corresponding utility comparison must be strict in at least one case.

We impose a simple regularity condition on the data (X, A, μ, P) before establishing this result.

Condition 1 *(X, A, μ, P) is **regular** if:*

1. *There exist $a_j, a_k \in A$ and $f, g \in \mathcal{F}(\mu)$ such that $\min\left[P^f(a_j), P^g(a_k)\right] > 0$.*

2. *Given* $a_j, a_k \in A$ *such that* $\min\left[P^f(a_j), P^g(a_k)\right] > 0$ *for some* $f, g \in \mathcal{F}(\mu)$*, there exists* $m \in \{1, \ldots, M\}$ *and* $n \in \{1, \ldots, N\}$*, such that,*

$$\frac{\sum_{\{f \in \mathcal{F} | f(a_m) = x_n\}} \mu(f) P^f(a_j)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(a_j)} \neq \frac{\sum_{\{f \in \mathcal{F} | f(a_m) = x_n\}} \mu(f) P^f(a_k)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(a_k)}.$$

The first regularity condition rules out the trivial case in which only one action is chosen. The second rules out cases in which two distinct actions cause precisely the same rational updating on the prize probabilities associated with all actions. With these assumptions, the following "No Improving Actions Switches" (NIAS) Lemma establishes the precise observable restrictions associated with existence of an optimal framing representation.

Lemma 1 (NIAS) *Regular* (X, A, μ, P) *has an optimal framing representation if and only if there exists* $U : X \to \mathbb{R}$ *satisfying the* ***NIAS inequalities****: for all* $j, k \in \{1, .., M\}$*,*

$$\sum_{f \in \mathcal{F}} \mu(f) P^f(a_j) U(f(a_j)) \geq \sum_{f \in \mathcal{F}} \mu(f) P^f(a_j) U(f(a_k)),$$

with at least one inequality strict.

A proof of this Lemma can be found in the Appendix.

The real bite of this result is that the condition is necessary. The fact that the NIAS conditions are sufficient for existence of an optimal framing representation is essentially immediate. Technically, it follows if one identifies a *single* subjective state with each action choice. However, one might imagine that if there are many subjective states in which a given action is taken, then it might be possible to correspondingly enrich the class of data sets consistent with the model. In fact this is not so: no matter how one enriches the state space, one cannot expand the empirical reach of the model.

6.4 NIAS and Framing Effects

The NIAS Lemma provides a complete characterization of all framing effects that are consistent with the model. It establishes that the optimal framing model has a simple general test, which corresponds to non-emptiness of the feasible set for a linear program.[8] The set of permissible

[8]Although not comparable because they are based on choices from budget constraints, Afriat [1967] similarly provided a set of data-defined linear inequalities such that a solution to the inequalities exists if and only if a non-satiated utility function exists that rationalizes the data.

framing effects is readily characterized in simple cases. Caplin and Martin [2011] provide a complete analysis of the two prize, two action case, and illustrate the important role that the prior μ plays in determining how restrictive are these inequalities. As demonstrated in section 3, these inequalities also provide a way to recover the set of utility functions in the presence of framing effects.

7 What Does and Does Not the Model Cover?

While the NIAS Lemma provides a mathematical test of consistency with our general model of framing effects, it may not be immediate whether existing models of limited or incorrect perception are consistent with our approach – in part because our separation of actions and prizes in framing is novel, and in part because these models are rarely stated in terms of their falsifiable implications. In this section, we examine a wide variety of models and see how they compare to our general model.

7.1 Excluded Models: Optimal Choice Violated

In Salant and Rubinstein [2008] , DMs are modeled as searching the first J options according to attentional ordering O and choosing the best option among those that have been observed, which generates several framing effects. Note that with this mode of behavior, the chosen object is always among those that have been explicitly searched. The same feature of choosing always within the searched set characterizes the alternative-based search model of Caplin and Dean [2011].

As a rule of behavior, choosing exclusively among the searched options can be inconsistent with maximization of expected utility when expectations are rational. Hence it is not consistent with our model. This is illustrated by example in section 3, in which identifying an item or items of low value in a given search made it optimal in our model to choose outside the searched set. This is quite general. If the prior μ allows all prizes to be found anywhere in the list, and search happens to reveal that the worst possible items are in the top J positions, any choice outside the searched set will yield higher expected utility than does any searched option.

7.2 Excluded Models: Rational Expectations Violated

There are many learning models in which beliefs never converge to rational expectations. For example, Esponda [2008] develops a model in which beliefs are based only on information concerning the choices that are actually made. Steiner and Stewart [2008] show how beliefs can be incorrect due to the presence of contagion effects. Schwartzstein [2012] models a continuous process of learning based on partial attention where beliefs have persistent errors. As it stands, our model is inconsistent with any such form of incomplete learning.

Settings in which rational expectations is most credible involve familiar environments in which the DM has learned through a process of trial and error the results of *any* available action in *any* perceptual state. As is often the case, this assumption is easiest to justify as the end result of an unmodeled and unobserved process of experimentation which has now concluded.

7.3 Included Models: Strategic Analogs

Our framework can be interpreted as a one player game of incomplete information against nature. Chance assigns the player a type t, which is composed of the frame the player faces and their subjective state,

$$t = (f, s) \in \mathcal{F}(\mu) \times S.$$

The player knows the joint likelihood of frames and subjective states and the actual subjective state they are in, but not the frame they face, which is reflected in the information sets presented in figure 1. The probability of being of type (f, s) given subjective state s is determined by Bayes' rule,

$$\Pr(f, s) = \frac{\mu(f)\pi^f(s)}{\sum_{g \in \mathcal{F}(\mu)} \mu(g)\pi^g(s)}.$$

Clearly, having a consistent belief system is analogous to holding rational expectations, and choices in Bayesian Nash equilibrium are analogous to optimal choices in the corresponding optimal framing representation.

Given this interpretation of the model, it is natural to develop strategic analogs of our model. To some extent, this development is under way. In highly complementary work, Bergemann and Morris [2011] establish that mutual NIAS inequalities characterize Bayesian Nash equilibria in a natural strategic setting. While our approaches differ in that we treat the IDS as the primitive and they treat utility functions as the primitive, the fact that we derive analogous inequalities is striking. Certainly it suggests that strategic models of framing based on imperfect perception may be worth exploring.

Our framework also has a conceptual link to the fully cursed equilibrium of Eyster and Rabin [2005]. Consider once again a player who knows the joint likelihoods and the subjective state they are in, but not the frame they face. However, suppose that the player believes incorrectly that frames create a stochastic action-prize mapping instead of a deterministic action-prize mapping, and that for a given subjective state, this stochastic action-prize mapping is the same for all frames and corresponds to the average chance of each action-prize being selected by nature. In this case, even though a player has incorrect beliefs, they are never contradicted by reality. The constraint that reality places on misperception in this example is similar to that produced by rational expectations in our model. In fact these seemingly distinct formulations may not be separable in the data.

There are also conceptual similarities between our framework and the analogous thinking model of Jehiel [2005]. Consider now a player who knows both the frame and their subjective state, but who only keeps track of the average connection of actions to prizes in each subjective state, not the actual connection of actions to prizes for each frame. This would be reasonable if there were too many frames practically to track. In this example, choices in Jehiel's model would be precisely as in our model with incomplete perception of the frame.

7.4 Included Models: Rational Inattention

Rational expectations is a standard assumption in many models of rational inattention (e.g. Sims [2003], Gabaix [2012], Woodford [2012]). In particular, Sims [2003] and Woodford [2012] allow for randomness in perception, just as do we, and similarly insist that beliefs are updated appropriately given the realized state of perception. While current models of rational inattention allow for a richer underlying state of the world than do we in our analysis of framing effects, they are more restrictive in their treatment of perceptual costs. Rather than treating these as unobservable, this literature makes strong assumptions on the perceptual cost function connected with the literature on entropy.

8 Concluding Remarks

We develop and apply an optimizing model that covers a wide variety of framing effects. This model can be used to study the long run impact of policy changes that impact the manner in which options are presented. In order for policy makers to analyze how best to frame a given set of options, they must be aware that their policy may change behavior by changing expectations (Muth [1961] and Lucas [1972]). Our formulation is particularly suitable for such purposes since it captures optimal responses to changes in expectations about the possible frames. Caplin and Martin [2012] develop just such a model to capture the impact of various possible default policies.

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10 Appendix

NIAS Lemma: Regular (X, A, μ, P) has an optimal framing representation (OFR) if and only if there exists $U : X \to \mathbb{R}$ satisfying the **NIAS inequalities**: for all $j, k \in \{1, .., M\}$,

$$\sum_{f \in \mathcal{F}} \mu(f) P^f(a_j) U(f(a_j)) \geq \sum_{f \in \mathcal{F}} \mu(f) P^f(a_j) U(f(a_k)),$$

with at least one inequality strict.

Proof. Sufficiency: We pick $\bar{U} : X \to [0, 1]$ satisfying the NIAS inequalities. We introduce a specific subjective state $\bar{s}^j$ per action $a_j \in A$ that is taken with strictly positive probability,

$$\bar{s}^j_{mn} = \frac{\sum_{\{f \in \mathcal{F}(\mu) | f(a_m) = x_n\}} \mu(f) P^f(a_j)}{\sum_{\{f \in \mathcal{F}(\mu)\}} \mu(f) P^f(a_j)},$$

and set,

$$\bar{\pi}^f(\bar{s}^j) = P^f(a_j) > 0,$$

for each such state, and for all $f \in \mathcal{F}(\mu)$. By regularity of (X, A, μ, P) note that there exist at least two possible actions, a_j and a_k. Regularity also implies that for any two such actions there exist $m \in \{1, \ldots, M\}$ and $n \in \{1, \ldots, N\}$ such that,

$$\frac{\sum_{\{f \in \mathcal{F} | f(a_m) = x_n\}} \mu(f) P^f(a_j)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(a_j)} \neq \frac{\sum_{\{f \in \mathcal{F} | f(a_m) = x_n\}} \mu(f) P^f(a_k)}{\sum_{f \in \mathcal{F}} \mu(f) P^f(a_k)}.$$

Hence the corresponding subjective states $\bar{s}^j$ and $\bar{s}^k$ are distinct, $\bar{s}^j \neq \bar{s}^k$, enabling us to define $\bar{C}(\bar{s}^j) = a_j$ without ambiguity on $s \in S(\bar{\pi})$. We define $\bar{C}(s)$ on $s \in S/S(\bar{\pi})$ to be any utility maximizing action. Finally, we define the perceptual cost function $K : \Pi^{RE}(\mu) \to \mathbb{R}_+$ to satisfy,

$$\bar{K}(\pi) = \begin{cases} 1 \text{ if } \pi = \bar{\pi}; \\ 0 \text{ if } \pi \neq \bar{\pi}. \end{cases}$$

Note that Data Matching holds by the definition of $\bar{C}$. That $\bar{\pi} \in \Pi^{RE}(\mu)$ follows from direct substitution,

$$\bar{s}^j_{mn} = \frac{\sum_{\{f \in \mathcal{F}(\mu) | f(a_m) = x_n\}} \mu(f) \bar{\pi}^f(\bar{s}^j)}{\sum_{\{f \in \mathcal{F}(\mu)\}} \mu(f) \bar{\pi}^f(\bar{s}^j)},$$

for all $m \in \{1, \ldots, M\}$, $n \in \{1, \ldots, N\}$, and $\bar{s}^j \in S(\pi)$. Perceptual Optimality follows since the utility advantage assigned to $\bar{\pi}$ over all alternative strategies exceeds the range of the utility function.

To establish Non-Triviality, select actions a_j and a_k for which the NIAS inequality is ever strict for $\bar{U}: X \rightarrow \mathbb{R}$. Substitution of $\bar{\pi}^f(\bar{s}^j) = P^f(a_j) > 0$ into the NIAS condition and re-organization by prize reveals,

$$\sum_{n=1}^{N} \sum_{\{f \in \mathcal{F}(\mu) | f(a_j)=x_n\}} \mu(f)\bar{\pi}^f(\bar{s}^j)\bar{U}(x_n) > \sum_{n=1}^{N} \sum_{\{f \in \mathcal{F}(\mu) | f(a_k)=x_n\}} \mu(f)\bar{\pi}^f(\bar{s}^j)\bar{U}(x_n).$$

Substitution in light of $\bar{\pi} \in \Pi^{RE}(\mu)$ reveals,

$$\sum_{n=1}^{N} \bar{s}^j_{jn}\bar{U}(x_n) > \sum_{n=1}^{N} \bar{s}^j_{kn}\bar{U}(x_n),$$

as required. Precisely analogous logic with weak inequalities replacing the above strict inequalities establishes Final Decision Optimality. Note that $\bar{s}^j \in S(\bar{\pi}) \Longrightarrow P^f(a_j) > 0$, whereupon substitution of $\bar{\pi}^f(s^j) = P^f(a_j)$ into the NIAS inequalities and reorganization by prize and substitution in light of $\bar{\pi} \in \Pi^{RE}(\mu)$ reveals,

$$\sum_{n=1}^{N} \bar{s}^j_{jn}\bar{U}(x_n) \geq \sum_{n=1}^{N} \bar{s}^j_{kn}\bar{U}(x_n),$$

for all $j, k \in \{1, .., M\}$, as required.

Necessity: A direct review of the above logic shows that identifying a utility function that satisfies the NIAS inequalities is not only sufficient for an OFR, but also necessary for an OFR in which there is only one subjective state per action taken with strictly positive probability. The full result follows from the observation that if we identify an OFR with more than one subjective state for one or more action choices, then there must exist an OFR with only one subjective state for each action choice: allowing for multiple states does not introduce new Ideal Data Sets which an OFR exists.

Consider an arbitrary OFR $(\tilde{K}, \tilde{C}, \tilde{U}, \tilde{\pi})$ of (X, A, μ, P). Given any action $a_j \in A$ such that $P^f(a_j) > 0$ some $f \in \mathcal{F}(\mu)$, let $S^j(\tilde{\pi}) \subset S(\tilde{\pi})$ be the finite set of possible states $\tilde{s}^{j,p} \in S(\tilde{\pi})$ such that,

$$\tilde{C}(\tilde{s}^{j,p}) = a_j,$$

for $1 \leq p \leq \left|S^j(\tilde{\pi})\right|$ (all possible statesin which a_j is chosen). Suppose now that set cardinalities are such that $\left|S^j(\tilde{\pi})\right| > 1$ for some such j. To prove necessity, we define a distinct OFR $(\bar{K}, \bar{C}, \bar{U}, \bar{\pi})$ of (X, A, μ, P) such that $\left|S^j(\bar{\pi})\right| = 1$ all such j.

The utility function is unchanged, $\bar{U} = \tilde{U}$. With regard to the states, given any action $a_j \in A$ such that $P^f(a_j) > 0$ some $f \in \mathcal{F}(\mu)$, we define a new subjective state, $\bar{s}^j_{mn}$, as the appropriately averaged version of the states in $S^j(\tilde{\pi})$,

$$\bar{s}^j_{mn} = \frac{\sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \sum_{\{f \in \mathcal{F}(\mu) | f(a_m)=x_n\}} \mu(f) \tilde{\pi}^f(\tilde{s}^{j,p})}{\sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \sum_{\{f \in \mathcal{F}(\mu)\}} \mu(f) \tilde{\pi}^f(\tilde{s}^{j,p})}.$$

In complementary fashion, define $\bar{C}(\bar{s}^j) = a_j$ and define $\bar{\pi}$ so that $\bar{s}^j$ is perceived in place of all $\tilde{s}^{j,p}$,

$$\bar{\pi}^f\left(\bar{s}^j\right) = \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \tilde{\pi}^f(\tilde{s}^{j,p}),$$

all $f \in \mathcal{F}(\mu)$. Again define $\bar{C}(s)$ on $s \in S/S(\bar{\pi})$ to be any utility maximizing action. Finally, define perceptual costs to satisfy,

$$\bar{K}(\pi) = \begin{cases} 1 \text{ if } \pi = \bar{\pi}; \\ 0 \text{ if } \pi \neq \bar{\pi}. \end{cases}$$

We now confirm that $(\bar{K}, \bar{C}, \bar{U}, \bar{\pi})$ provide an OFR of (X, A, μ, P). Note first that Data Matching is satisfied by construction. That $\bar{\pi} \in \Pi^{RE}(\mu)$ follows from rearranging summations and substituting,

$$\begin{aligned} \bar{s}^j_{mn} &= \frac{\sum_{\{f \in \mathcal{F} | f(a_m)=x_n\}} \mu(f) \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \tilde{\pi}^f(\tilde{s}^{j,p})}{\sum_{\{f \in \mathcal{F}\}} \mu(f) \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \tilde{\pi}^f(\tilde{s}^{j,p})} \\ &= \frac{\sum_{\{f \in \mathcal{F}(\mu) | f(a_m)=x_n\}} \mu(f) \bar{\pi}^f(\bar{s}^j)}{\sum_{\{f \in \mathcal{F}(\mu)\}} \mu(f) \pi^f(\bar{s}^j)}, \end{aligned}$$

for all $m \in \{1, \ldots, M\}$, $n \in \{1, \ldots, N\}$, and $\bar{s}^j \in S(\pi)$. It is then immediate that Perceptual Optimality is satisfied.

Final Decision Optimality is established by noting that it survives under convex combinations. To see this, select some state $s^j \in S(\bar{\pi})$. Note from the construction that this implies that $P^f(a_j) > 0$ some $f \in \mathcal{F}(\mu)$. Since $(\tilde{K}, \tilde{C}, \tilde{U}, \tilde{\pi})$ form an OFR of (X, A, μ, P), this in turn implies that,

$$\sum_{n=1}^{N} \tilde{s}^{j,p}_{jn} \bar{U}(x_n) \geq \sum_{n=1}^{N} \tilde{s}^{j,p}_{kn} \bar{U}(x_n),$$

for all $k \in \{1, \ldots, M\}$ and $1 \leq p \leq S^j(\tilde{\pi})$. Substitution in light of rational expectations and addition across p implies,

$$\sum_{n=1}^{N} \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \sum_{\{f \in \mathcal{F} | f(a_j)=x_n\}} \mu(f) \tilde{\pi}^f(\tilde{s}^{j,p}) \bar{U}(x_n) \geq \sum_{n=1}^{N} \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \sum_{\{f \in \mathcal{F} | f(a_k)=x_n\}} \mu(f) \tilde{\pi}^f(\tilde{s}^{j,p}) \bar{U}(x_n).$$

Upon substitution for $\bar{s}^j$ in terms of $\tilde{s}^{j,p}$, this reduces to,

$$\sum_{n=1}^{N} \bar{s}^j_{jn} \bar{U}(x_n) \geq \sum_{n=1}^{N} \bar{s}^j_{kn} \bar{U}(x_n),$$

for all $\{1, \ldots, M\}$, establishing Final Decision Optimality.

Finally, we turn to non-triviality. Since $(\tilde{K}, \tilde{C}, \tilde{U}, \tilde{\pi})$ form an OFR of (X, A, μ, P), we know that there exists $j, k \in \{1, \ldots, M\}$ and $\tilde{s} \in S(\tilde{\pi})$ such that,

$$\sum_{n=1}^{N} \tilde{s}_{jn} \bar{U}(x_n) > \sum_{n=1}^{N} \tilde{s}_{jn} \bar{U}(x_n).$$

In light of Final Decision Optimality, it is WLOG to assume that $\tilde{s} \in S^j(\tilde{\pi})$. With this combined with all other weak inequalities, we conclude in light of rational expectations that,

$$\sum_{n=1}^{N} \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \sum_{\{f \in \mathcal{F} | f(a_j) = x_n\}} \mu(f) \tilde{\pi}^f(\tilde{s}^{j,p}) \bar{U}(x_n) > \sum_{n=1}^{N} \sum_{p=1}^{\left|S^j(\tilde{\pi})\right|} \sum_{\{f \in \mathcal{F} | f(a_k) = x_n\}} \mu(f) \tilde{\pi}^f(\tilde{s}^{j,p}) \bar{U}(x_n).$$

At this point substitution as above for $\bar{s}^j$ in terms of $\tilde{s}^{j,p}$ reveals the desired strict inequality,

$$\sum_{n=1}^{N} \bar{s}^j_{jn} \bar{U}(x_n) > \sum_{n=1}^{N} \bar{s}^j_{kn} \bar{U}(x_n).$$

■